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CURRENTLY USED SMOOTHING AND DIFFERENTIATION PROCEDURES FOR OBTAINING VELOCITIES AND ACCELERATIONS AND THEIR EFFECT ON DISPERSION (U)

By

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(U) ABSTRACT

The reasons for the selection of the smoothing and differentiation formulas, which are currently used in calculation of smooth missile positions, velocities and accelerations, are studied. The formulas are described in detail and their effect is illustrated. Approximate values of the noise level in the smooth data are provided and the magnitude of systematic error due to these procedures is estimated.

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(U) LIST OF SYMBOLS

$_n$	Unsmoothed posit... Y, or ... subscript ... t... particular point
$_L$	Unsmoothed position X, Y, or Z at last point
$_n$	Smoothed position X, Y, or Z
$_n$	Unsmoothed velocit... X, Y, or Z calculated from smoothed positions.
$_n$	Unsmoothed acceleration X, Y, or ... calculated from smoothed positions.
$_n$	Smoothed velocity X ..., or Z calculated from smooth... positions.
$_n$	Smoothed acceleration X, Y, or Z calculated from smoothed positions.
t	Time interval of input data.
$_{co}$	Time of chamber pressure drop following ... off signal
$_{CP}$	Time of chamber pressure level-off following T_{co}.
$_L$	Time of last point in input data.
T_{to}	Missile liftoff time.
T_O	Time of first point in input data.

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SECTION I. (S) INTRODUCTION

In the analysis of missile test flights, velocities and accelerations are the bases of many other calculations. One method of determining velocities and accelerations is by numerical differentiation of position data. The position data may be obtained from one of several types of instrumentation. The data contain random errors of observation and reduction as well as systematic errors. It is usually necessary to smooth the data to obtain realistic numerical derivatives. The numerical smoothing and differentiation procedures have undergone considerable evolutionary change as a result of experience with varied instrumentation, missile systems, and flight paths. The complexity of the procedures has increased greatly. Questions have frequently arisen concerning the present smoothing and differentiation procedures and the reasons for using these procedures. This report provides some answers by giving some insight into the general problem of smoothing and differentiation and by description of the currently used procedures.

In analyzing smoothing and differentiation procedures it is desirable to have some means of estimating the dispersion of noise in positions, velocities, and accelerations. A method has been devised for doing this and is described briefly. A method is also described and applied for determining the systematic errors introduced by the smoothing and differentiation procedures.

SECTION II. (S) SMOOTHING AND DIFFERENTIATION PROCEDURES CURRENTLY IN USE

1. Development

The smoothing procedures now in use in the Data Reduction B[illegible] generally use moving arc smoothing formulas. In this operation a curve is fitted to an arbitrary number of points which are usually serial at a fixed time interval and represent a segment of a time series. One or more points, usually the central point, is adjusted to [illegible] to the fitted curve. Then the curve fit formula is shifted along the time series so that one new point is added to the set and one old point at the other end of the series is removed. The fitting and adjustment procedure is then reapplied to the new set, leading to the adjustment of a point adjacent to the previously adjusted point. This procedure may be continued over a major portion of a time series. This point-by-point moving arc smoothing reduces the discontinuities due to end effects to a minimum by distributing them among all the intervals.

The early smoothing procedures employed involved unweighted polynomial approximation by least squares and orthogonal polynomial formulas. Later it was found that the smoothing formulas derived by L. S. Dede[illegible] (Ref 1) were convenient and gave superior results. The goal of a smoothing formula is to increase the smoothness of the data without excessi[illegible]

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increasing the adjustments necessary to achieve this smoothness. The smoothness and adjustments may be measured in terms of the magnitude of th. n^{th} order differences and the magnitude of the residuals.

he ..locities and accel... s calculated by numerical differ- io. frequently showed osci..ations of considerable a.p itude at..n or reduct.on of these oscillations, which were considered is..c, was required. The amplitude of these oscillations increased .y with an increasing degree of the smoothing formula. Thus it was ible to use as low a degree as possible without causing gross .ti. of the original data. It was found that a degree lower than d c. ld not be used with the point spreads that were being considered d degree Dederick smoothing formulas of increasing point spread app.ied to actual data. In that way a high degree of local hne.. could be achieved while the data still contained very distinct o .at.ons of considerable amplitude. It was apparent that the .at..ns could be reduced by increasing the point spread of the a d..ree smoothing so as to encompass several oscillations. Thus .rm..a would not be able to follow the individual oscillations and th...fore reduce their amplitudes. Our smoothing formula was .ed to cover a 20-second time interval in order to ac.omplish this io. in the oscillations. One-tenth of a second time steps ... s a . point smoothing formu.. This large number of points ... d the calculation time on a mac.ne appreciable and the build- ... . a-off errors might be appreciable also. The difficulty was a..eviat. by using a 101 point, second degree smoothing formula which used ever. second point in the sequence. A further improvement in the lc. smoothness of the velocities and accelerations was achieved by .g a .econd pass smoothing of forty-one points and second degree

Thi. smoothing procedure has .e disad.antage of not being able to .ve any physical fluctuatio. having a .eriod and amplitude similar less than that of the osci..ations. The characteristic Mach one ba..e is of sufficient period and amplitude to remain distinc. .ver, the characteristic engine cutoff pattern would be grossly .te. .v this smoothing procedure. In order to preserve the .te..stic engine cutoff pattern, the point spread of the smoothing .re..d in steps as the time of cutoff is approached. After cutoff the point spread is increased in steps back to that of the general .ia. Although this permits the preservation of the general .cteristic pattern it leaves both noise and oscillations in the data the vicinity of cutoff. Smoother values of accelerations are .esirable for use in other calculations. Therefore a second degree polynomial is fitted to the ten seconds of acceleration data immediately preceding cutoff. This polynomial is evaluated to get smooth accelerations for the five seconds immediately preceding cutoff. Another second degree polynomial is fitted to the ten seconds of acceleration data immediately following the chamber pressure level-off following cutoff. This polynomial is evaluated to get smooth accelerations for the five seconds immediately following chamber pressure level-off .me.

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Special procedures are also used for smoothing and differentiation at the beginning and at the end of the time series. These involve the use of shorter point spreads and asymmetric formulas

A general purpose smoothing and differentiation program utilizing Dedekind coefficients was developed in the Test Data Processing Section. This program was used in some of our studies. It was possible to select any point spread up through twenty-five and any degree up through four. A number of programs utilizing higher point spreads were prepared by Milton L. Whigham of the Test Data Processing Section for use in our studies.

Obviously the procedures could be greatly improved if the oscillations could be kept from developing. It has been discovered that some contribution to the oscillations may be due to roundoff exceeding the relative accuracy of the data. This phenomenon has been studied and reported (Ref. 2). It may be possible to eliminate this source of oscillations. It has also been established that some contribution to the oscillations is due to the smoothing of random noise. This phenomenon has also been studied and reported (Ref. 3). This latter oscillation source cannot be easily eliminated since it is due only to the randomness of the noise and the sampling rate. Other sources of oscillations in the various types of tracking instrumentation also exist.

Description

The present smoothing and differentiation procedures are programmed for the IBM No. 709. The input to the program is trajectory position data calculated at a fixed time interval. The program consists of two main parts. In the first part the position data are smoothed and first and second derivatives are calculated at each time step using these smoothed positions. In the second part of the program the calculated velocities and accelerations are smoothed and a second degree curve fit is used to obtain smooth accelerations near cutoff time.

a. Initial equations of the first part

$$\bar{U}_0 = \dot{\bar{U}}_0 = \ddot{\bar{U}}_0 = 0 \qquad \text{when } t_0 \leq t_{to}$$

$$\bar{U}_0 = \frac{1}{5}(3U_0 + 2\bar{U}_1 + \bar{U}_2 - \bar{U}_4) \qquad \text{when } t_0 > t_{to} \tag{2}$$

$$\dot{\bar{U}}_0 = \dot{\bar{U}}_1 - (\dot{\bar{U}}_2 - \dot{\bar{U}}_1) \qquad \text{when } t_0 > t_{to}$$

$$\ddot{\bar{U}}_0 = \ddot{\bar{U}}_1 - (\ddot{\bar{U}}_2 - \ddot{\bar{U}}_1) \qquad \text{when } t_0 > t_{to}$$

$$\bar{U}_1 = \frac{1}{5}(3U_1 + 2\bar{U}_2 + \bar{U}_3 - \bar{U}_5) \tag{5}$$

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$$\dot{\bar{U}}_1 = \frac{\bar{U}_2 - \bar{U}_0}{2\Delta t} \tag{6}$$

$$\ddot{\bar{U}}_1 = \frac{\bar{U}_2 - 2\bar{U}_1 + \bar{U}_0}{\Delta t^2} \tag{7}$$

$$\bar{U}_2 = \sum_{i=-2}^{+2} C_i \, U_{i+2} \tag{8}$$

where the C's may be found in Column A of Table 1.

$$\dot{\bar{U}}_2 = \frac{\bar{U}_3 - \bar{U}_1}{2\Delta t} \tag{9}$$

$$\ddot{\bar{U}}_2 = \frac{\bar{U}_3 - 2\bar{U}_2 + \bar{U}_1}{\Delta t^2} \tag{10}$$

$$\bar{U}_n = \sum_{i=-3}^{+3} C_i \, U_{n+i} \qquad \text{when } 3 \leq n \leq 14 \tag{11}$$

where the C's may be found in Column B of Table 1

$$\dot{\bar{U}}_n = \frac{\bar{U}_{n+1} - \bar{U}_{n-1}}{2\Delta t} \qquad \text{when } 3 \leq n \leq 14 \tag{12}$$

$$\ddot{\bar{U}}_n = \frac{\bar{U}_{n+1} - 2\bar{U}_n + \bar{U}_{n-1}}{\Delta t^2} \qquad \text{when } 3 \leq n \leq 14 \tag{13}$$

$$\bar{U}_n = \sum_{i=-15}^{+15} C_i \, U_{n+i} \qquad \text{when } 15 \leq n \leq 24 \tag{14}$$

where the C's may be found in Column D of Table 1.

For $\dot{\bar{U}}_n$ and $\ddot{\bar{U}}_n$ when $15 \leq n \leq 24$ see Equations (12) and (13).

$$\bar{U}_n = \sum_{i=-25}^{+25} C_i \, U_{n+i} \qquad \text{when } 25 \leq n \leq 49 \tag{15}$$

[redacted]

where the C's may be found in Column F of Table 1.

For $\dot{\bar{U}}_n$ and $\ddot{\bar{U}}_n$ when $25 \leq n \leq 49$ see Equations (12) and (13).

$$\bar{U} = \sum_{i=-50}^{+50} C_i \, U_{n+i} \qquad \text{when } 50 \leq n \leq 99 \qquad (16)$$

where the C's may be found in Column G of Table 1.

For $\dot{\bar{U}}_n$ and $\ddot{\bar{U}}_n$ when $50 \leq n \leq 99$ see Equations (12) and (13).

b. <u>General equations of the first part</u>

$$\bar{U}_n = \sum_{i=-50}^{+50} C_i \, U_{n+2i} \qquad \text{when } 100 \leq n \leq (t_{co} - 101\,\Delta t) \qquad (17)$$

re .e C's may be found in Co.umn G of Table 1.

$$\dot{\bar{U}} = \frac{\bar{U}_{n+2} - \bar{U}_{n-2}}{4\Delta t} \qquad ([illegible])$$

$$\ddot{\bar{U}} = \frac{\bar{U}_{n+2} - 2\bar{U}_n + \bar{U}_{n-2}}{2\Delta t^2} \qquad ([illegible])$$

c. <u>Cutoff equations of the first part</u>

or $\bar{U}_n$, $\dot{\bar{U}}_n$, $\ddot{\bar{U}}_n$:

when $t_{co} - 100\,\Delta t \leq n \leq t_{co} - 51\,\Delta t$ see Equations (16), (12), (1

when $t_{co} - 50\,\Delta t \leq n \leq t_{co} - 26\,\Delta t$ see Equations (15), (12), (13)

$$\bar{U}_n = \sum_{i=-10}^{+10} C_i \, U_{n+i} \qquad \text{when } t_{co} - 25\,\Delta t \leq n \leq t_{co} + 25\,\Delta t \qquad (20)$$

where the C's may be found in Column C of Table 1.

For $\dot{\bar{U}}_n$, $\ddot{\bar{U}}_n$ when $t_{co} - 25\,\Delta t \leq n \leq t_{co} + 25\,\Delta t$ see Equations (12), (13)

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For $\bar{U}_n$, $\dot{\bar{U}}_n$, $\ddot{\bar{U}}_n$:

when $t_{co} + 26\,\Delta t \le n \le t_{co}$ 50 Δt see Equations (15), (12), (13),

when $t_{co} + 51\,\Delta t \le n \le t_{co} + 100\,\Delta t$ see Equations (16), (12), (13),

when $t_{co} + 101\,\Delta t \le n \le t_L - 100\,\Delta t$ see Equations (17), (18), (19).

<u>Terminal equations of the first part</u>

For $\bar{U}_n$, $\dot{\bar{U}}_n$, $\ddot{\bar{U}}_n$:

when $t_L - 99\,\Delta t \le n \le t_L - 50\,\Delta t$ see Equations (16), (12), (13),

when $t_L - 49\,\Delta t \le n \le t_L - 25\,\Delta t$ see Equations (15), (12), (13),

when $t_L - 24\,\Delta t \le n \le t_L - 15\,\Delta t$ see Equations (14), (12), (13),

when $t_L - 14\,\Delta t \le n \le t_L - 3\,\Delta t$ see Equations (11), (12) (13).

$$\bar{U}_{\cdot} = \sum_{i=-2}^{+2} C_i\, U_{L-2+i} \tag{21}$$

[illegible]re [illegible] may be found in Column A of Table 1.

$$\dot{\bar{U}}_{\cdot} = \frac{\bar{U}_{L-1} - \bar{U}_{L-3}}{2\Delta t} \tag{22}$$

$$\ddot{\bar{U}}_{\cdot} = \frac{\bar{U}_{L-1} - 2\bar{U}_{L-2} + \bar{U}_{L-3}}{\Delta t^2} \tag{23}$$

$$\bar{U}_l = \frac{1}{5}\left(3U_{L-1} + 2\bar{U}_{L-2} + \bar{U}_{L-3} - \bar{U}_{L-5}\right) \tag{24}$$

$$\dot{\bar{U}}_l = \frac{\bar{U}_L - \bar{U}_{L-2}}{2\Delta t} \tag{25}$$

$$\ddot{\bar{U}}_{\cdot} = \frac{\bar{U}_L - 2\bar{U}_{L-1} + \bar{U}_{L-2}}{\Delta t^2} \tag{26}$$

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$$\bar{U}_L = \frac{1}{5}(3U_L + 2\bar{U}_{L-1} + \bar{U}_{L-2} - \bar{U}_{L-4}) \quad (27)$$

$$\dot{\bar{U}}_L = \frac{1}{12\Delta t}(3\bar{U}_{L-4} - 6\bar{U}_{L-3} + 36\bar{U}_{L-2} - 48\bar{U}_{L-1} + 25\bar{U}_L) \quad (28)$$

$$\ddot{\bar{U}}_L = \frac{1}{12\Delta t^2}(11\bar{U}_{L-4} - 56\bar{U}_{L-3} + 114\bar{U}_{L-2} - 104\bar{U}_{L-1} + 35\bar{U}_L) \quad (29)$$

scontinuities exist in the smooth data at the junction points twe the different smoothing formulas. These discontinuities resu ... ve wild derivatives at these points. Special treatment was necessary at these changeover points when seven point spread smoothing or greater was used. A maximum of four points of velocity and acceleration were replaced at each junction. These replacements were based on second degree curve fit through the previous seven points of velocity and acceleration. The method of least squares was used for the curve .

e. Junction point replacement equations

$$= A_0 + A_1 t + A_2 t^2$$

$$_n = B_0 + B_1 t + B_2 t^2$$

where A_0, A_1, A_2, B_0, B_1, B_2 are the coefficients obtained by the abov mentioned least squares curve fits.

f. General equations of the second part

$$\dot{\bar{U}}_n = \sum_{i=-20}^{+20} C_i \dot{\bar{U}}_{n+i} \quad \text{when } 20 \le n \le t_{co} - 51\,\Delta t \text{ and when } t_{CPL} + 51\,\Delta t \le n \le t_L - 20\,\Delta t \quad (32)$$

where C's may be found in Column E of Table 1.

$$\ddot{\bar{U}}_n = \sum_{i=-20}^{+20} C_i \ddot{\bar{U}}_{n+i} \quad \text{when } 20 \le n \le t_{co} - 51\,\Delta t \text{ and when } t_{CPL} + 51\,\Delta t \le n \le t_L - 20\,\Delta t$$

wher C's may be found in Column E of Table 1.

g. Cutoff equation of the second part

$$\ddot{\bar{U}}_n = A_0 + A_1 t + A_2 t^2 \quad \text{when } t_{co} - 50\,\Delta t \le n \le t_{co} \quad (34)$$

[redacted]

This second degree curve fit is obtained by applying the method of least squares to the one hundred points preceding t_{CO}.

$$- B_O + B_1 t \qquad t^2 \qquad \text{when } t_{CPL} \leq n \leq t_{CPL} + 50\,\Delta t \qquad (35)$$

This second degree curve fit is obtained by applying the method of least squares to the one hundred points following t_{CPL}.

The actual calculations in the program are carried out in two distinct passes through the data. The smoothing of the positions and the calculation of velocities and accelerations are done in the first pass. The application of the various formulas of the first pass to the particular parts of the trajectory is summarized in Figure 1. Details of the application of the smoothing formulas in the first pass are illustrated in Figures 3 through 5.

The smoothing of velocities and accelerations and the curve fitting of accelerations in the vicinity of cutoff are done in the second pass. The application of the various formulas of the second pass to the particular parts of the trajectory is summarized in Figure 2.

Effects

The significant factor concerning these smoothing and differentiation procedures is their effectiveness in producing smooth and realistic velocity and acceleration data. Figures 6 through 8 show velocities which were calculated by the current smoothing and differentiation program. The data used in the calculations were at a one-tenth second time interval whereas the data used in the graphs were selected at one second time intervals. Figures 9 through 13 show segments of the velocity data on a smaller scale in order to illustrate effectively the local smoothness. These data are at the one-tenth second time interval which was used in the calculations. Figures 14 through 16 show accelerations which were calculated by the current smoothing and differentiation program. The data used in the calculations were at one-tenth second time interval whereas the data used in the graphs were selected at one-second time intervals. Figures 17 through 21 show segments of the acceleration data on a smaller scale in order to illustrate effectively the local smoothness. These data are at the one-tenth second time interval which was used in the calculations.

It will be noted that some problem areas remain in these procedures. The discontinuities in accelerations at the end points of the curve fit data preceding and following cutoff represent a difficulty which needs further improvement. The noise and oscillations which remain in the acceleration data for the period of thrust decay represent another problem area. These difficulties are clearly manifested in Figures 20 and 21 and will be eliminated as time permits.

It is sometimes desirable to have a quantitative measure of the dispersion of the noise in order to compare the relative merits of different smoothing techniques. In order to estimate the dispersion of noise in our positions, velocities and acceleration a curve fitting program was used to fit these smooth data at successive time intervals. A second degree polynomial was fitted to each ten-second interval and residuals were calculated. The standard deviation of the residuals was calculated for each interval. It was assumed that the second degree polynomial was capable of following the general trend of the data over most of the ten-second intervals. It was also assumed that the second-degree polynomial was not capable of following the noise or other minor fluctuations in a ten-second interval. Therefore the standard deviation of the residuals should be a fair estimate of the noise level of the smoothed data. Figures 22 through 30 show these calculated standard deviations for the smoothed positions, velocities and accelerations. Generally the noise level in smooth UDOP positions is less than 2.0 meters, in smooth UDOP velocities is less than .07 meter per second, and in smooth UDOP accelerations is less than .02 meter per second per second.

The achievement of smoothness is of little value if it is attained at the expense of gross distortion of the original data. It would certainly not be feasible to use smoothing formulas which regularly produced systematic errors which exceed the noise level of the smoothed data. It was therefore desirable to determine the magnitude of systematic errors produced by our current smoothing and differentiation procedures. A synthetic trajectory program was used to generate smooth positions, velocities and accelerations representative of a typical missile flight. These smooth positions were then used as input to our current smoothing and differentiation program. These smoothed positions, velocities and accelerations were then differenced with the smooth positions, velocities and accelerations generated by the synthetic trajectory. The differences indicate systematic errors introduced by the smoothing and differentiation program. Figures 31 through 39 show these differences. As might be expected the differences only become appreciable at times of radical physical change such as main engine cutoff (157.77 seconds), vernier engine ignition (166.28 seconds), and vernier engine cutoff (176.29 seconds).

SECTION III (S) CONCLUSIONS

It may be concluded that the current smoothing and differentiation procedures are satisfactory for most parts of a typical missile test flight and for typical tracking instrumentation. The exceptions are the times of rapid physical change such as main engine cutoff, vernier engine ignition, and vernier engine cutoff. The attainment of equivalent accuracy at these times requires additional observation and special treatment. Investigation of these possibilities will proceed as time permits.

The noise level in the smooth UDOP positions is generally less than 2.0 meters. In the smooth UDOP velocities the noise level is generally less than .07 meter per second and in the smooth UDOP accelerations is generally less than .02 meter per second per second. The systematic errors introduced by the smoothing and differentiation procedures are generally less than the noise levels of the smooth data.

GENERAL DEDERICK COEFFICIENTS

i	C_i A	B	C	D	E	F	G
-50							-.00002743
-49							-.00010058
-48							-.00022983
-47							-.00041900
-46							-.00066634
-45							-.00096567
-44							-.00130722
-43							-.00167849
-42							-.00206506
-41							-.00245117
-40							-.00282041
-39							-.00315619
-38							-.00344223
-37							-.00366293
-36							-.00380373
-35							-.00385142
-34							-.00379429
-33							-.00362241
-32							-.00332768
-31							-.00290399
-30							-.00234725
-29							-.00165542
-28							-.00082850
-27							.00013152
-26							.00122070
-25						-.00032438	.00243325
-24						-.00108383	.00376162
-23						[illegible]	.00519666

i	C_i A	B	C		E	F	G
-22						-.00363836	.00672771
-21						-.00508725	.00834281
-20					-.00068479	-.00635536	.01002880
-19					-.00217926	-.00721713	.01177154
-18					-.00424676	-.00746965	.01355607
-17					-.00644783	-.00694741	.01536680
-16					-.00826372	-.00553220	.01718766
-15				-.00171211	-.00918903	-.00315853	[illegible]
-14				-.00500293	-.00879756	.00018451	.02079439
-13				-.00875514	-.00678499	.00445470	[illegible]
-12				.01150094	-.00299216	.00956047	[illegible]
-11				-.01186182	.00258781	.01536678	.02587324
-10			-.00557029	-.00882682	.00981543	.02170263	.02741519
-9			-.01345481	-.00189905	.01842084	.02836945	.02885725
-8			-.01761357	.00886228	.02802735	.03515033	.03018606
-7			-.01289565	.02287915	.03818004	.04181938	.03138929
-6			.00311907	.03914878	.04837719	.04815117	.03245574
-5			.02962788	.05637424	.05810267	.05392963	.03337550
-4			.06303804	.07311382	.06685749	.05895644	.03414001
-3		-.05874125	.09795617	[illegible]	.07418899	.06305848	.03474215
-2	-.07342657	.05874125	.12842263	.09954972	.07971656	.06609416	.03517629
-1	.29370629	.29370629	.14913596	.10695119	.08315262	.06795860	.03543841
0	.55944055	.41258741	.15646914	[illegible]	.08431822	[illegible]	[illegible]
+1	.29370629	.29370629	.14913596	.10695119	.08315262	.06795860	[illegible]
+2	-.07342657	.05874125	.12842263	.09954972	.07971656	.06609416	.03517629
+3		-.05874125	.09795617	[illegible]	.07418899	.06305848	.03474215
+4			.06303804	[illegible]	.06685749	.05895644	.03414001
+5			[illegible]	[illegible]	.05810267	.05392963	.03337550
+6			.00311907	.03914878	.04837719	.04815117	.03245574
+7			.01289565	.02287915	.03818004	.04181938	.03138929

TABLE 1 (Continued)

i	C_1 A	B	C	D	E	F	G
+8			-.01761357	.00886228	.02802735	.03515033	.03018606
+9			-.01345481	-.00189905	.01842084	.02836945	.02885725
+10			-.00557029	-.00882682	.00981543	.02170263	.02741519
+11				-.01186182	.00258781	.01536678	.02587324
+12				-.01150094	-.00299216	.00956047	.02424568
+13				-.00875514	-.00678499	.00445470	.02254753
+14				-.00500293	-.00879756	.00018451	.02079439
+15				-.00171211	-.00918903	-.00315853	.01900233
+16					-.00826372	-.00553220	.01718766
+17					-.00644783	-.00694741	.01536680
+18					-.00424676	-.00746965	.01355607
+19					-.00217926	-.00721713	.01177154
+20					-.00068479	-.00635536	.01002880
+21						-.00508725	.00834281
+22						-.00363836	.00672771
+23						-.00223633	.00519666
+24						-.00108383	.00376162
+25						-.00032438	.00243325
+26							.00122070
+27							.00013152
+28							-.00082850
+29							-.00165542
+30							-.00234725
+31							-.00290399
+32							-.00332768
+33							-.00362241
+34							-.00379429
+35							-.00385142
+36							-.00380373

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i	A	B	C	D	E	F	G
+37							-.00366293
+38							-.00344223
+39							-.00315619
+40							-.00282041
+41							-.00245117
+42							-.00206506
+43							-.00167849
+44							-.00130722
+45							-.[illegible]096567
+46							-.00066634
+47							-.0004[illegible]900
+48							-.00022983
+49							-.00010058
+50							-.00002743

[redacted]

(U) REFERENCES

1 Dederick, L. S., Construction and Selection of Smoothing Formulas. Report No. 863, Ballistic Research Laboratories; Aberdeen Proving Ground, Maryland. 1953

2. Beard, L. Neel and MacGowan, Roger A., Oscillations In Tracking Data Resulting from Roundoff. Report No. DC-TR-1-60, Army Ballistic Missile Agency, Computation Laboratory; Redstone Arsenal, Alabama. 1960

3. Beard, L. Neel and MacGowan, Roger A., Oscillatory Phenomena In Empirical Data Resulting from the Smoothing of Random Noise. Report No. DC-TR-3-60, Army Ballistic Missile Agency, Computation Laboratory; Redstone Arsenal, Alabama. 1960

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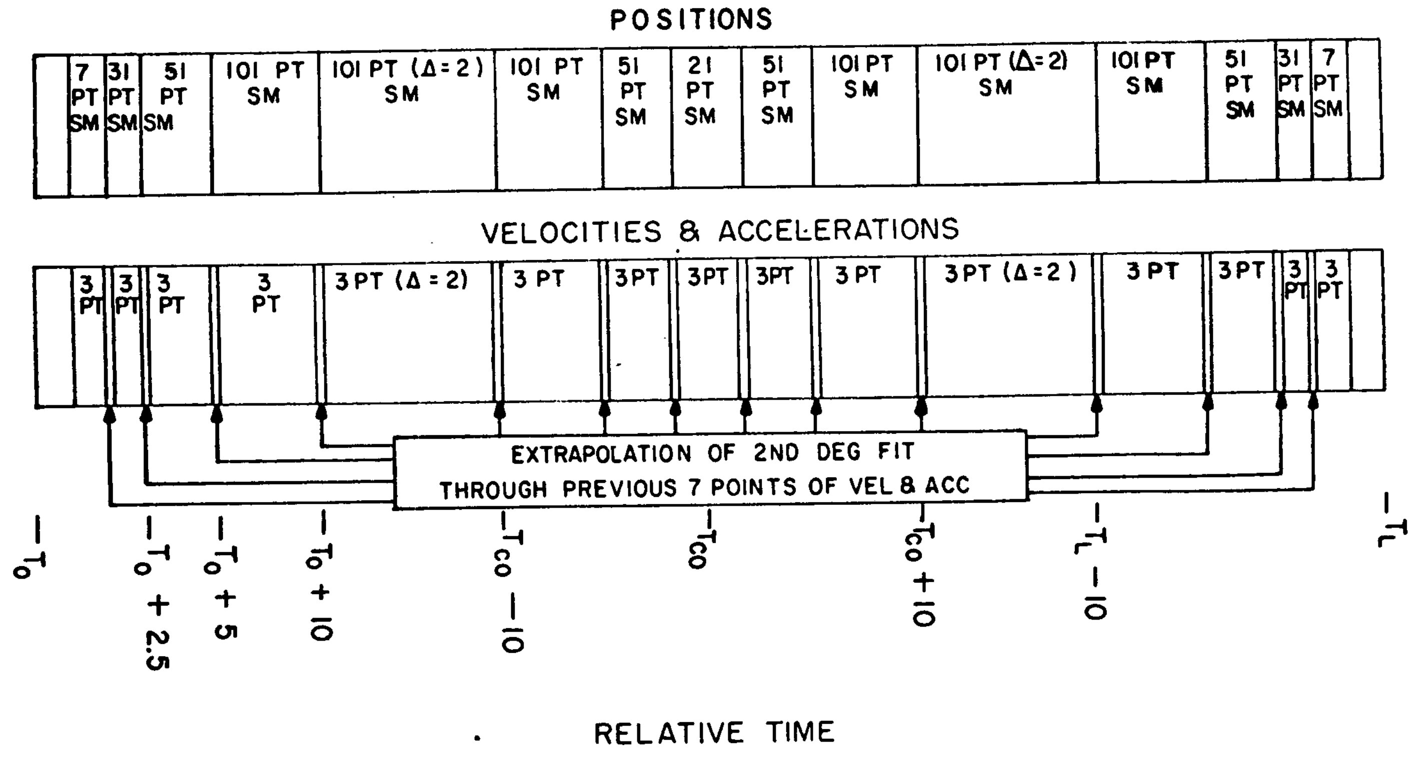

FIG.1 - FIRST PASS OF PROGRAM

[redacted]

[redacted]

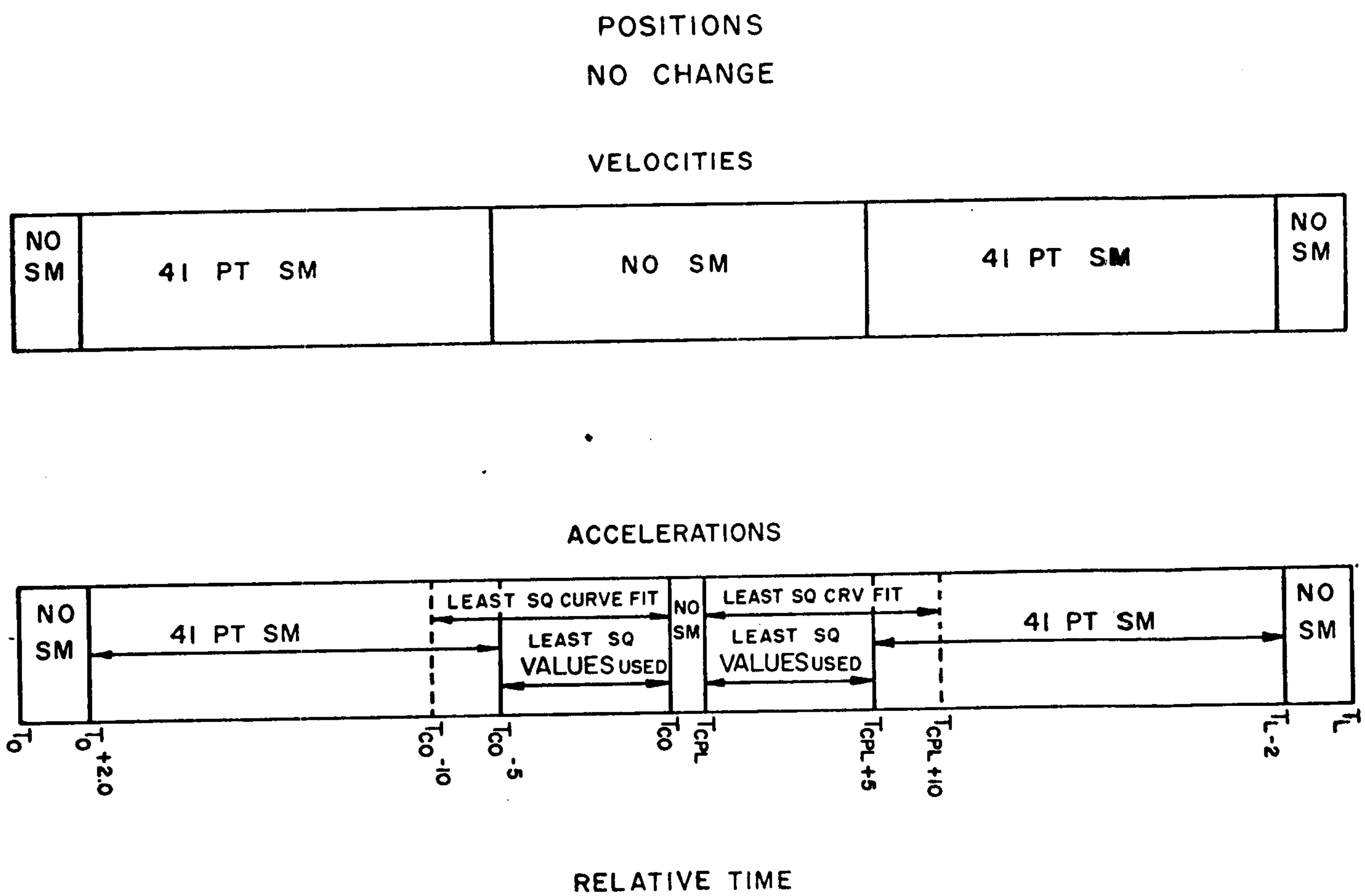

FIG. 2 - SECOND PASS OF PROGRAM

[redacted]

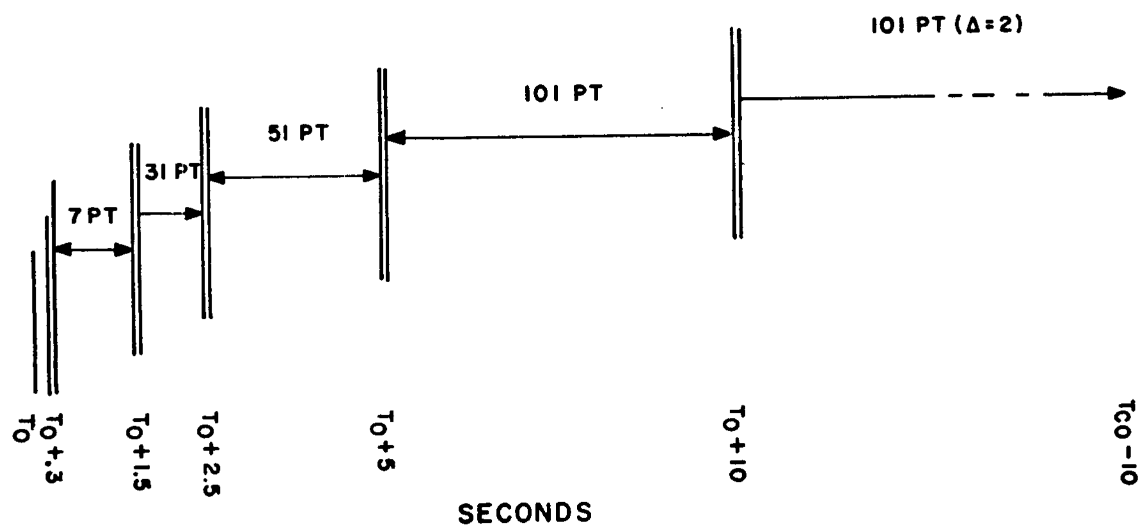

FIG. 3. INITIAL SECTION OF FIRST PASS

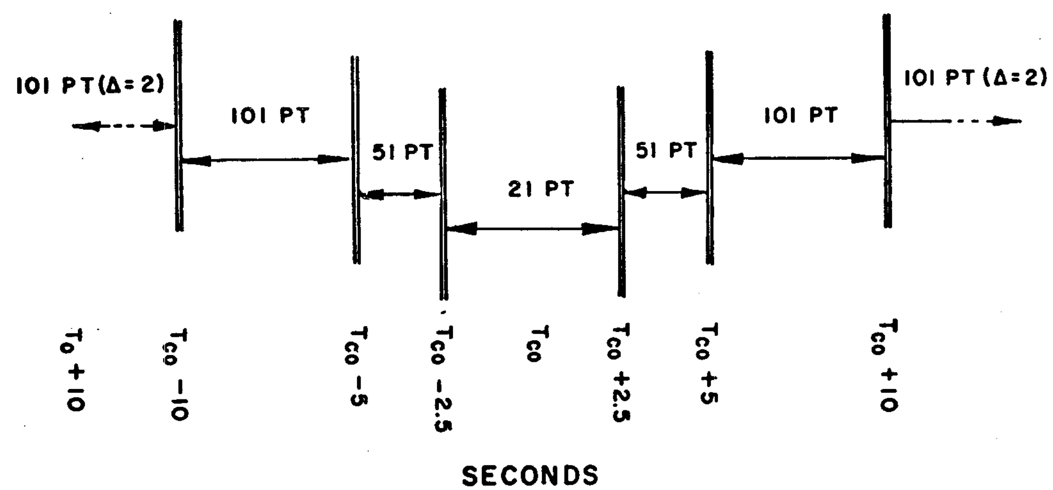

FIG. 4. CUTOFF SECTION OF FIRST PASS

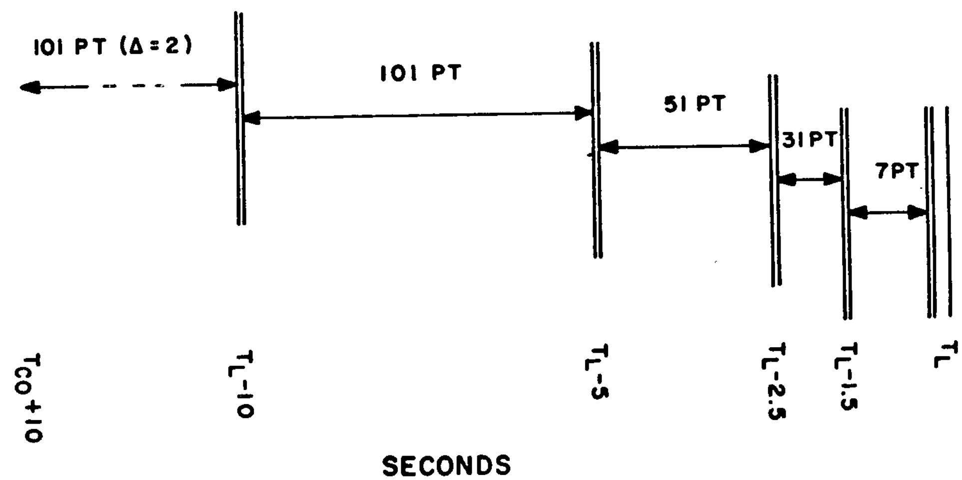

FIG. 5. FINAL SECTION OF FIRST PASS

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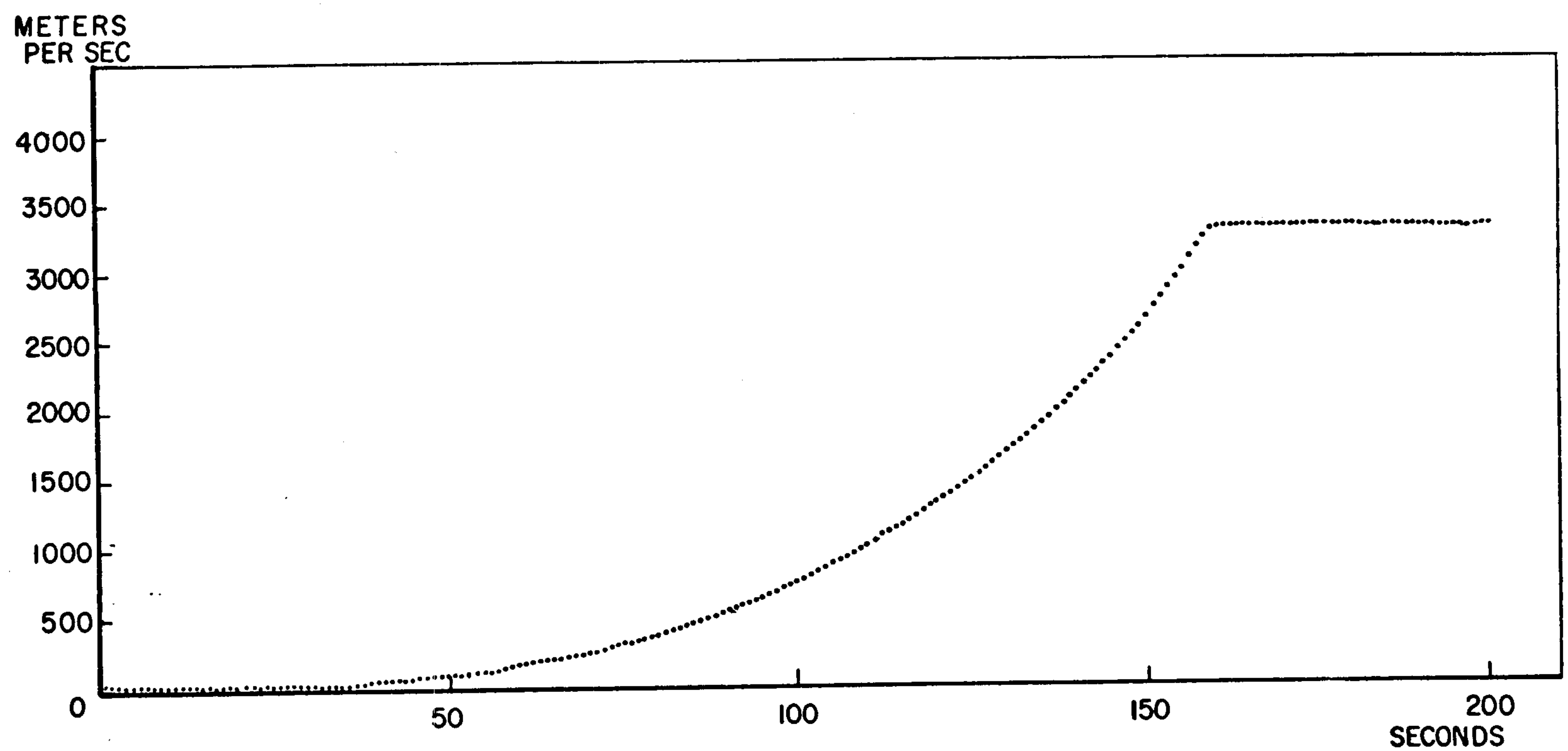

FIG 6. SMOOTH X VELOCITY UDOP (0 to 200 sec)

[redacted]

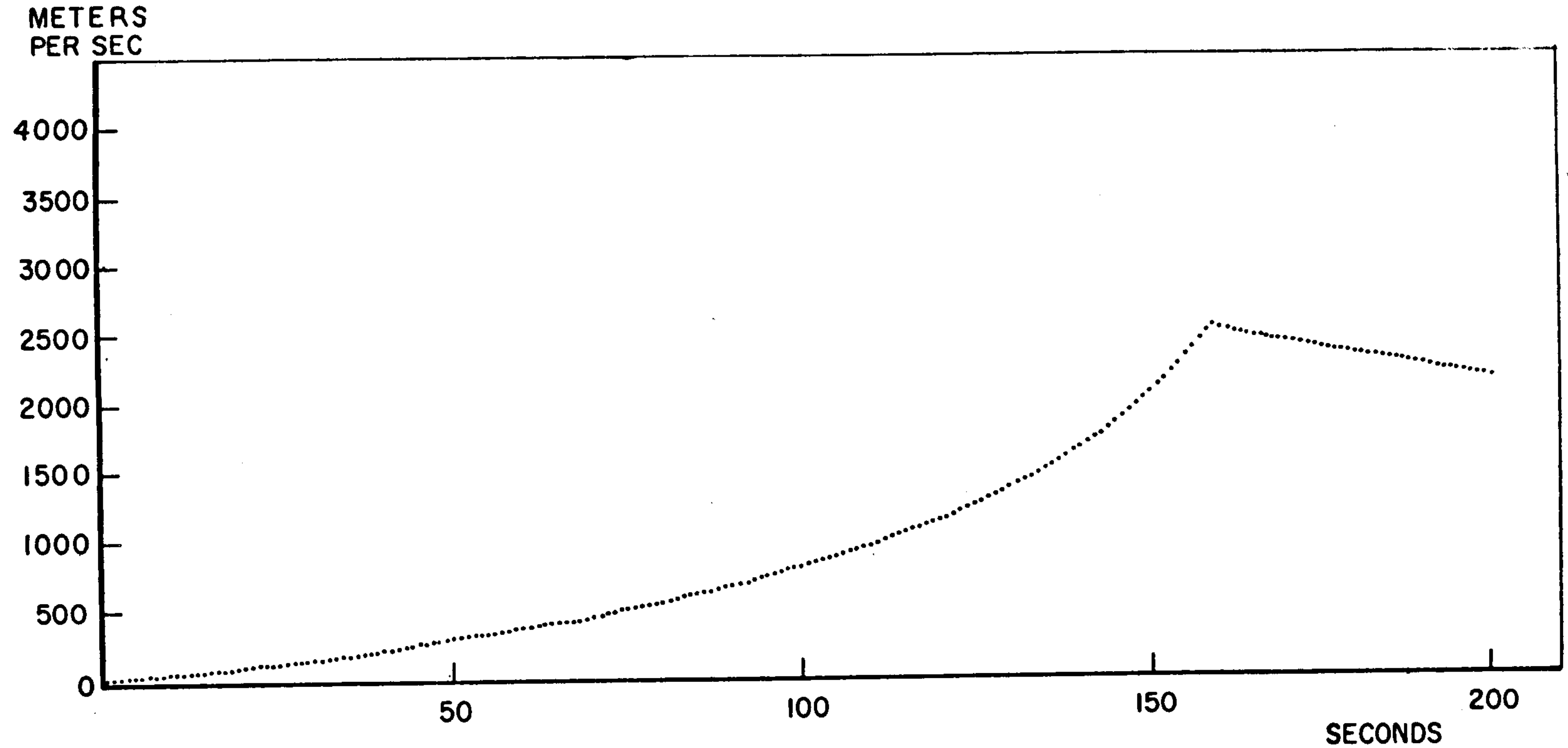

FIG 7. SMOOTH Y VELOCITY UDOP (0 to 200 sec)

[redacted]

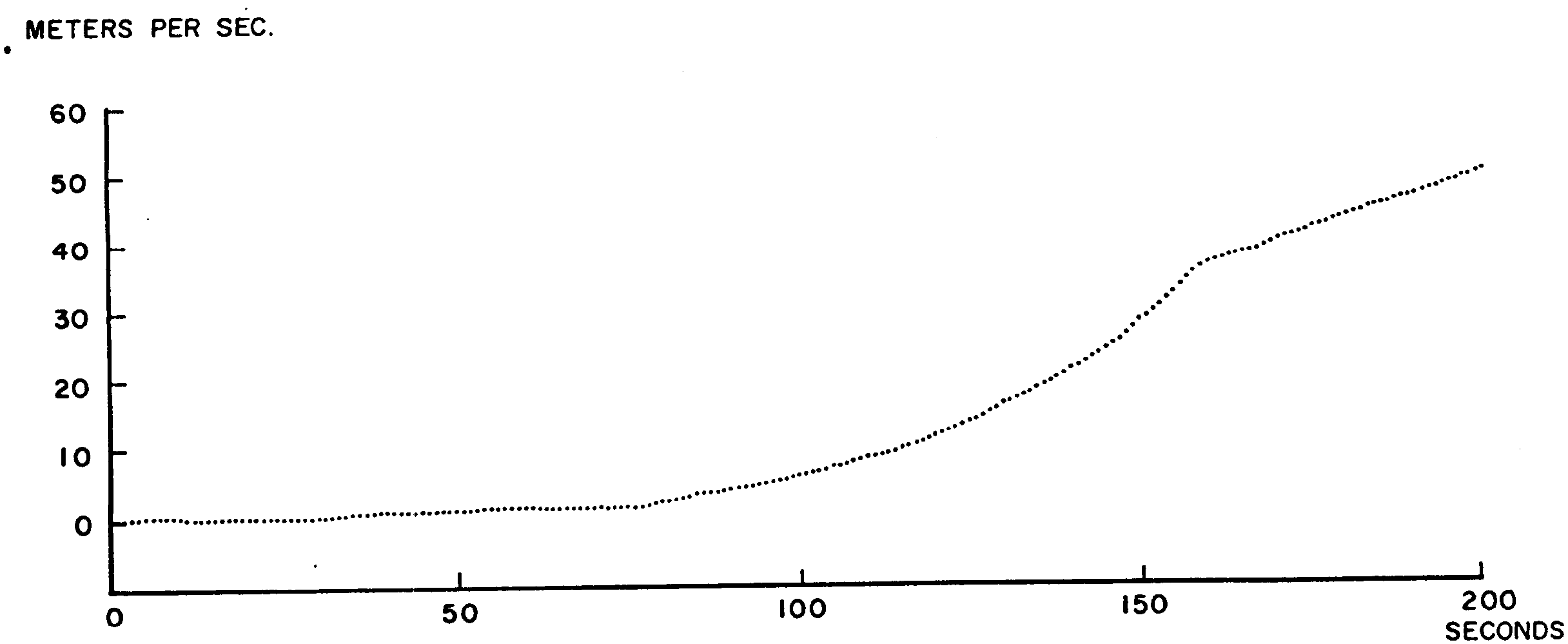

FIG. 8. SMOOTH Z VELOCITY UDOP (0 TO 200 SECONDS)

[redacted]

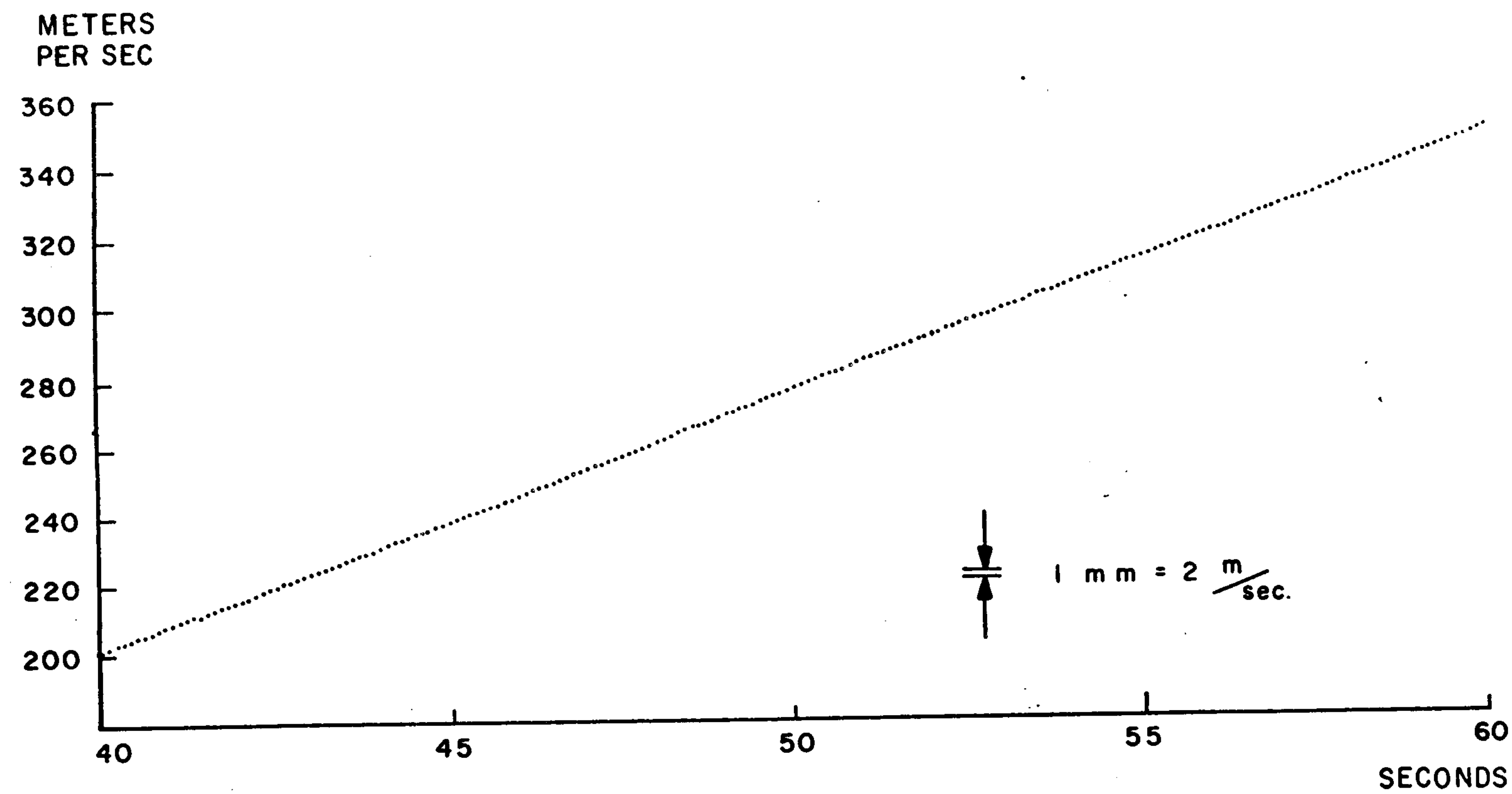

FIG. 9. SMOOTH Y VELOCITY UDOP (40 TO 60 SECONDS)

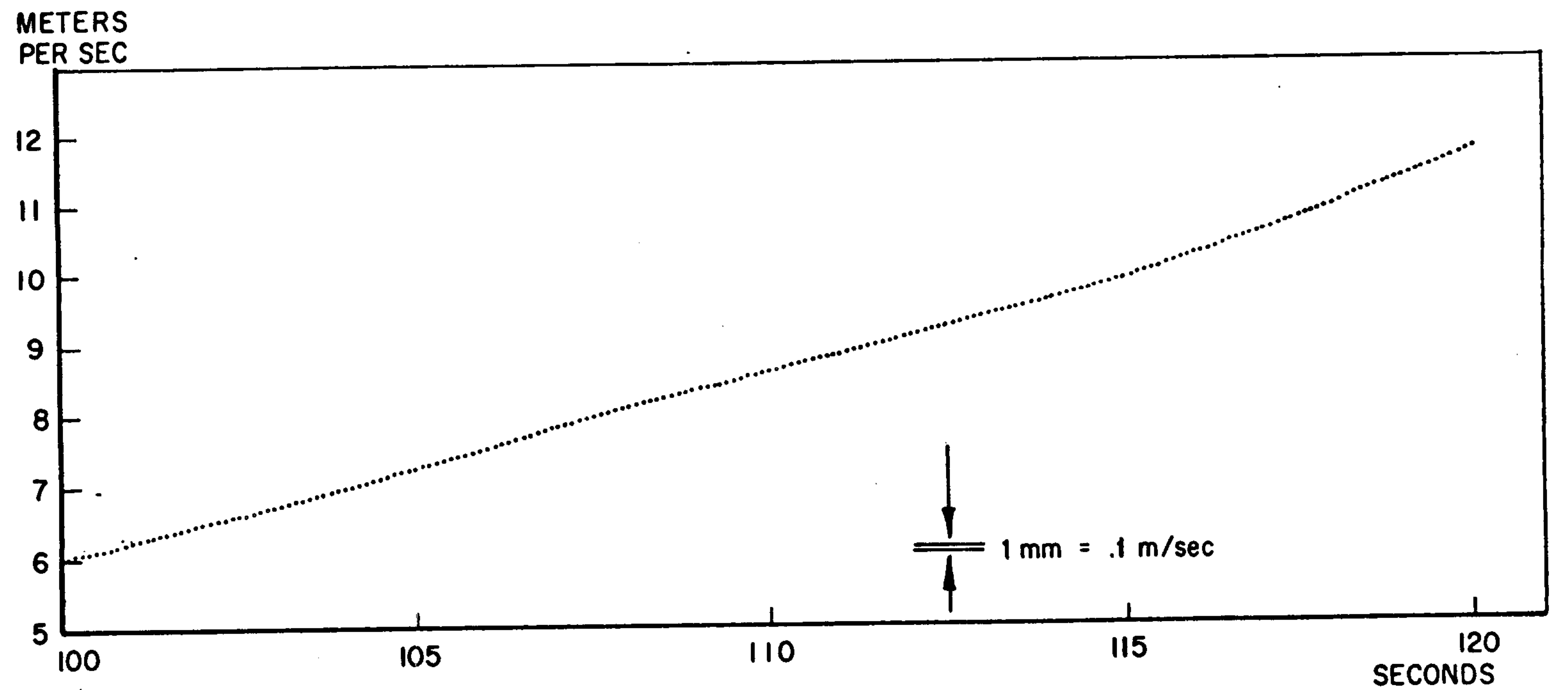

FIG 10. SMOOTH Z VELOCITY UDOP (100 to 120 sec)

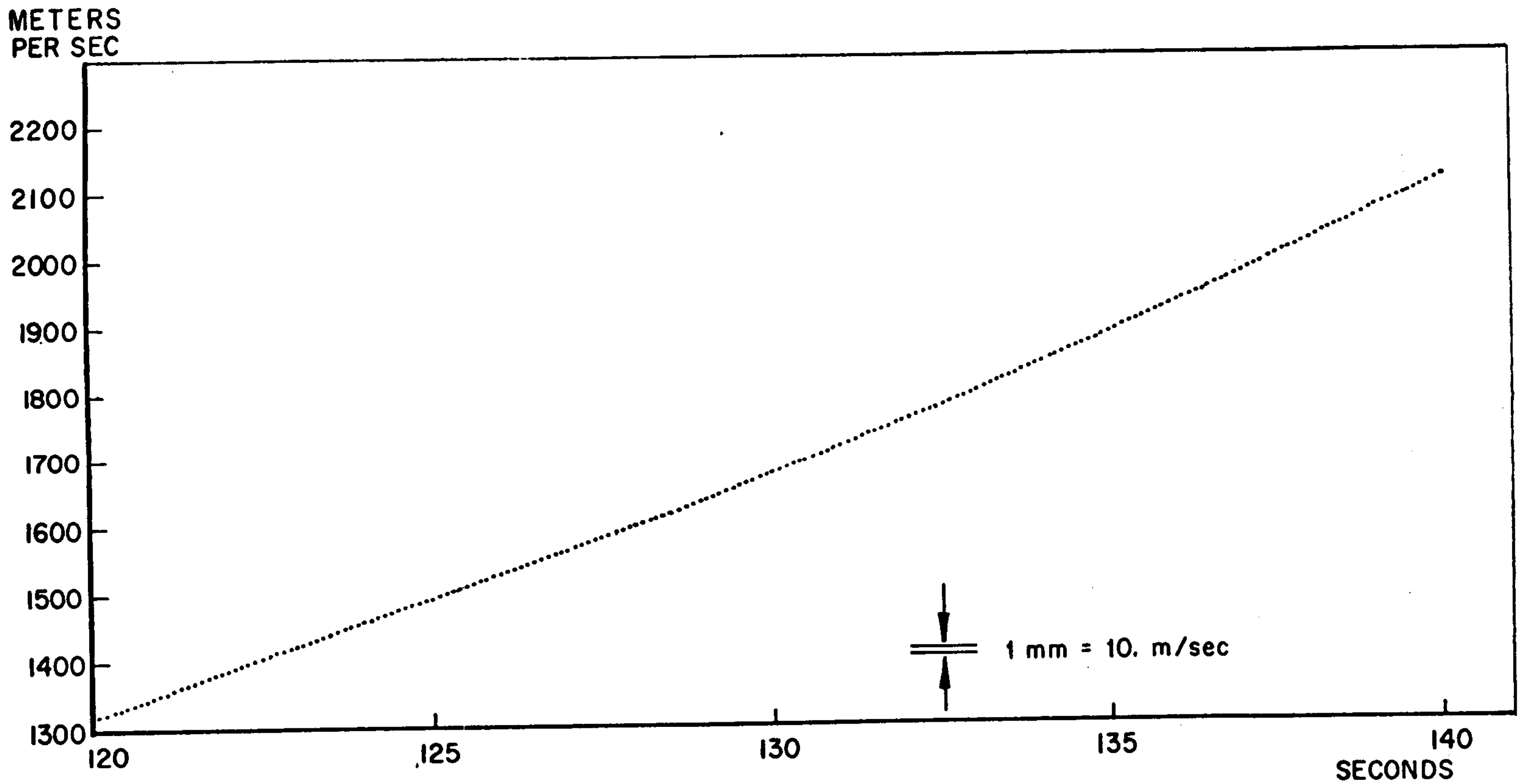

FIG 11. SMOOTH X VELOCITY UDOP (120 to 140 sec)

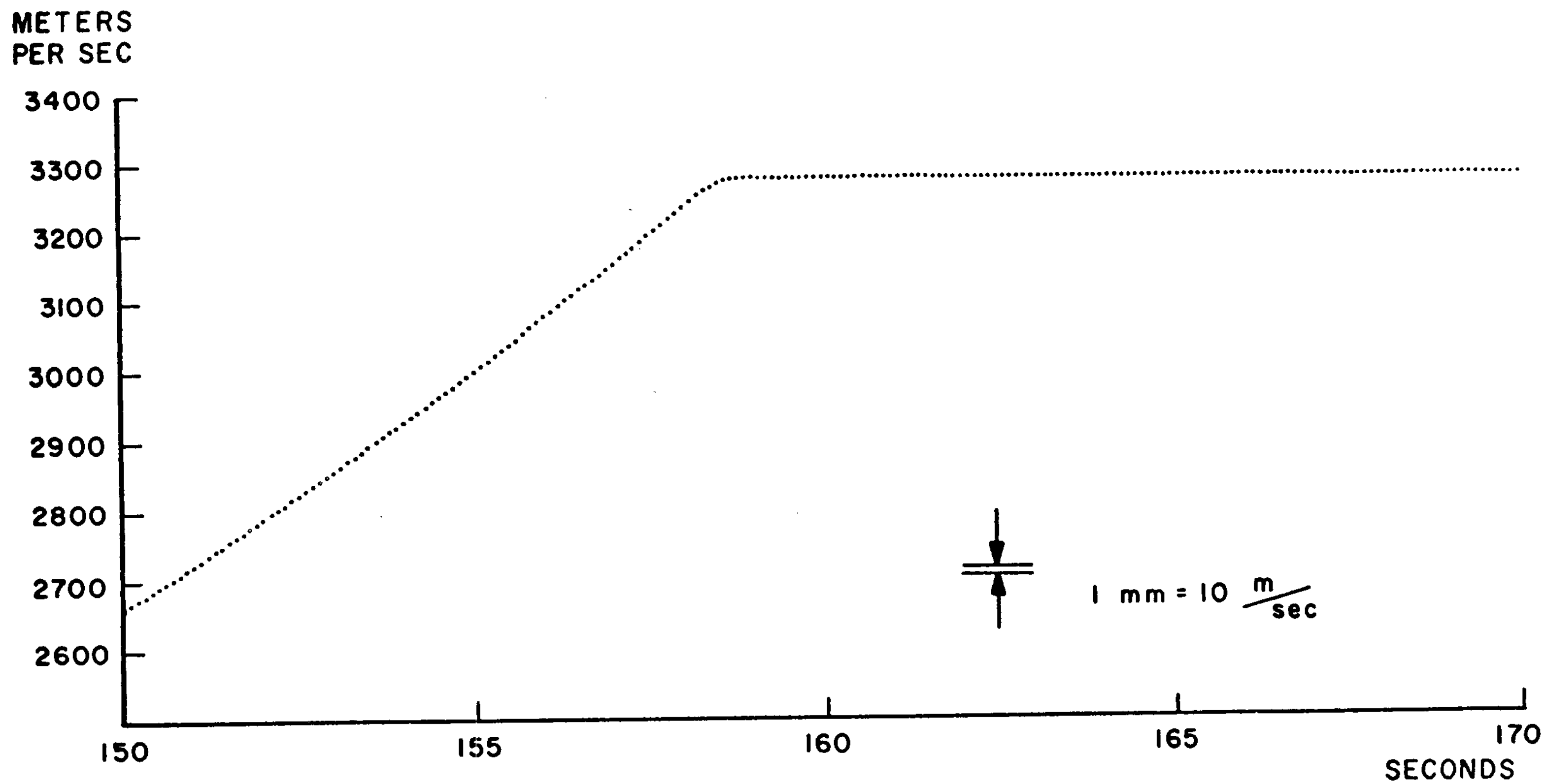

FIG. 12. SMOOTH X VELOCITY UDOP (150 TO 170 SECONDS)

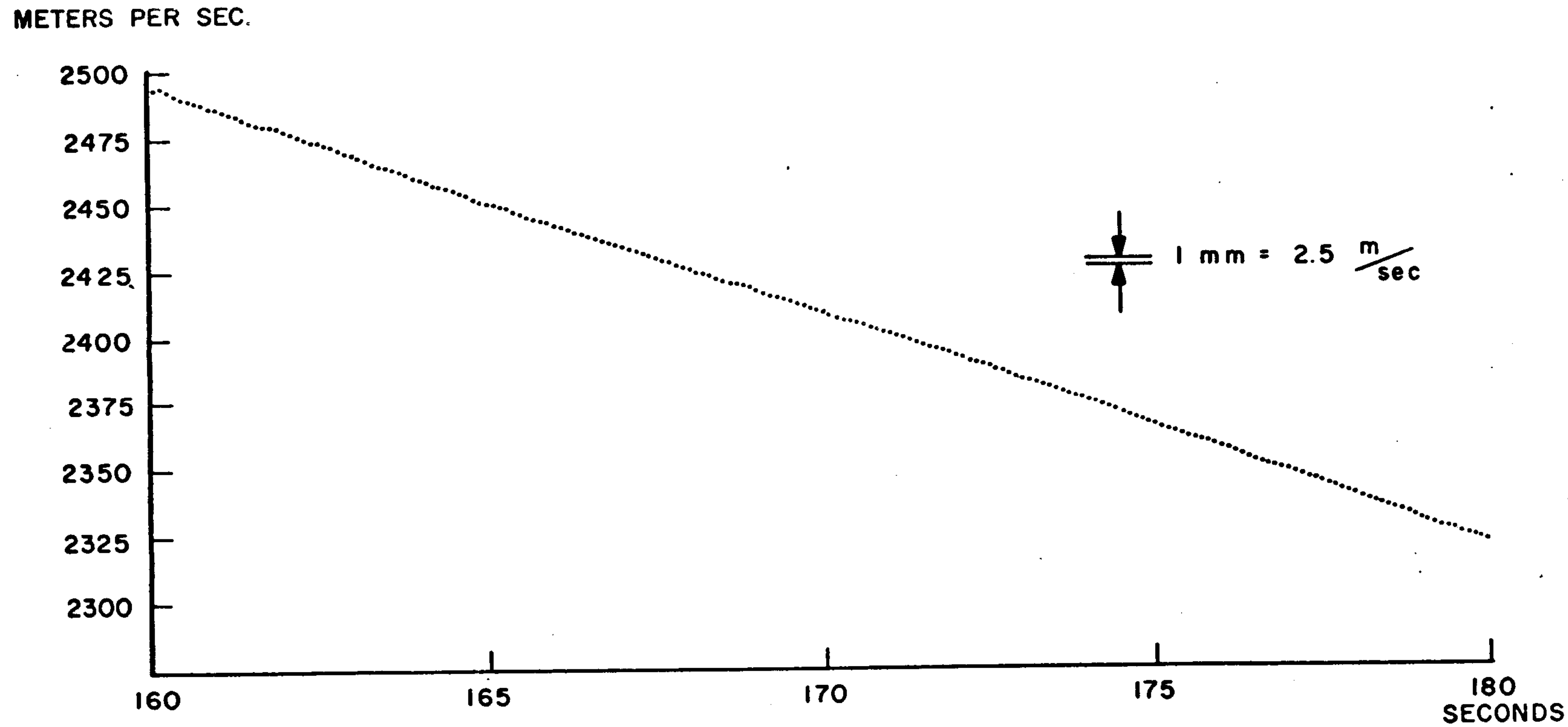

FIG. 13. SMOOTH Y VELOCITY UDOP (160 TO 180 SECONDS)

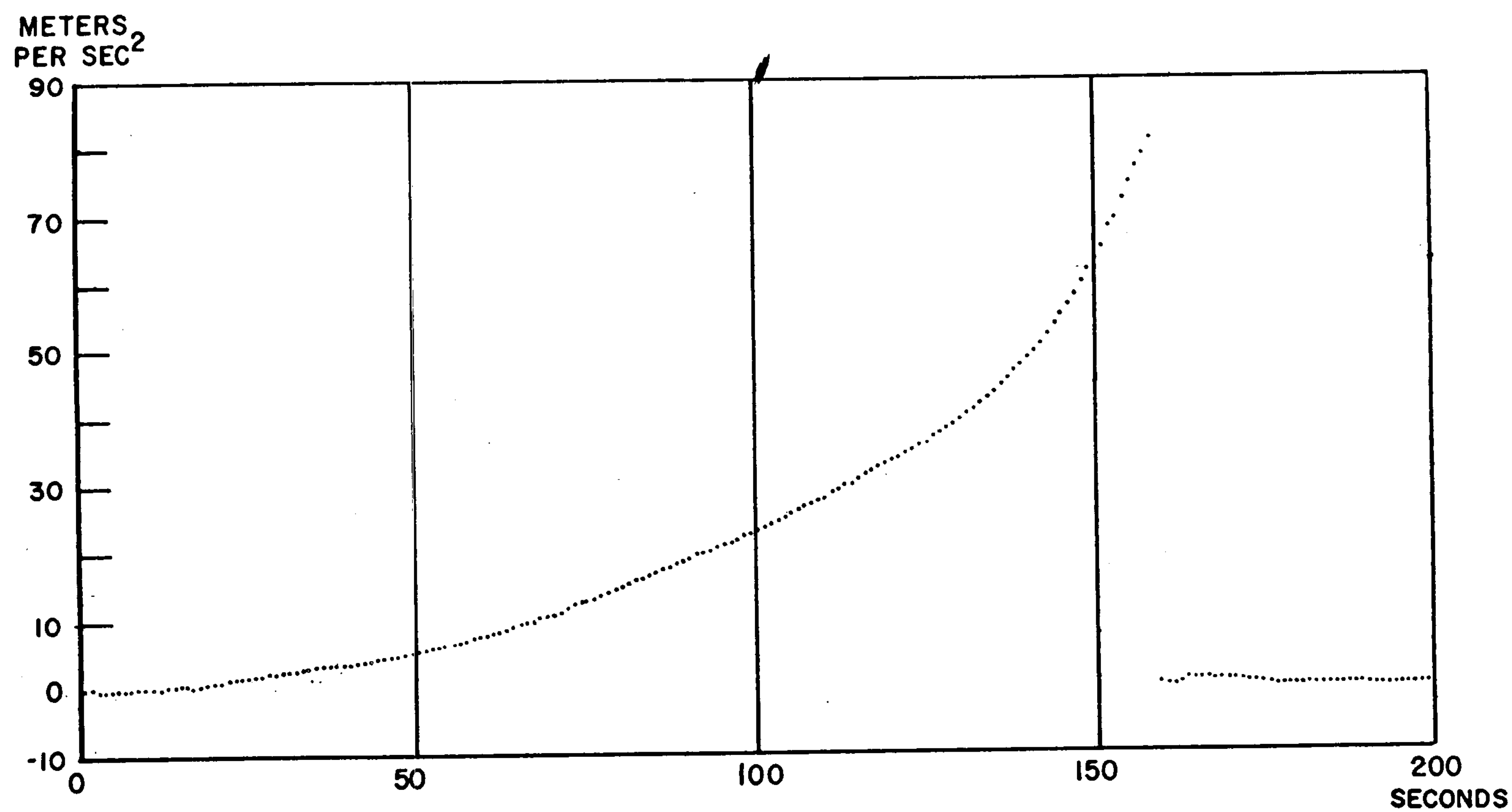

FIG 14 SMOOTH X ACCELERATION UDOP (0 TO 200 SECONDS)

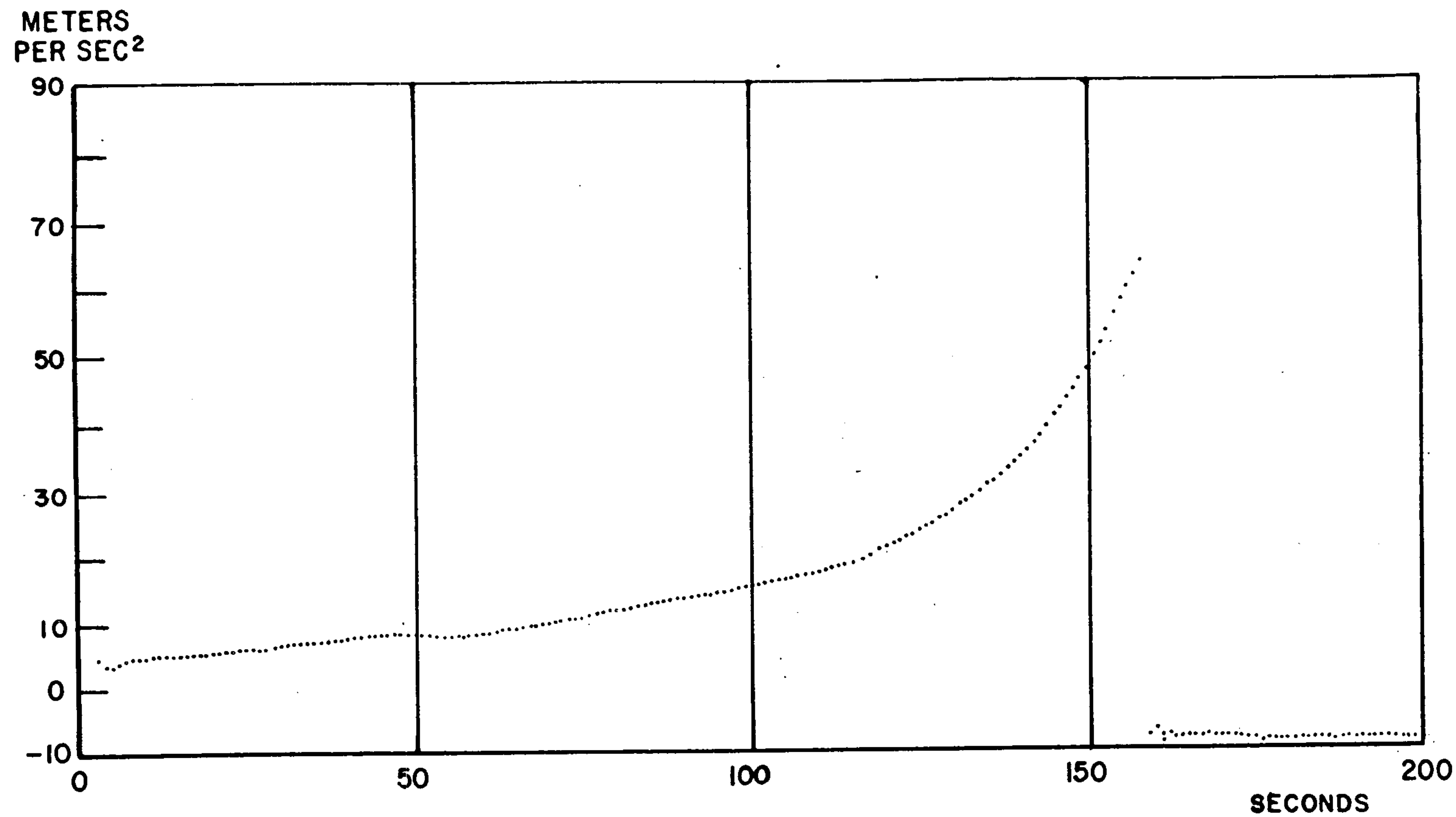

FIG 15 SMOOTH Y ACCELERATION UDOP (0 TO 200 SECONDS)

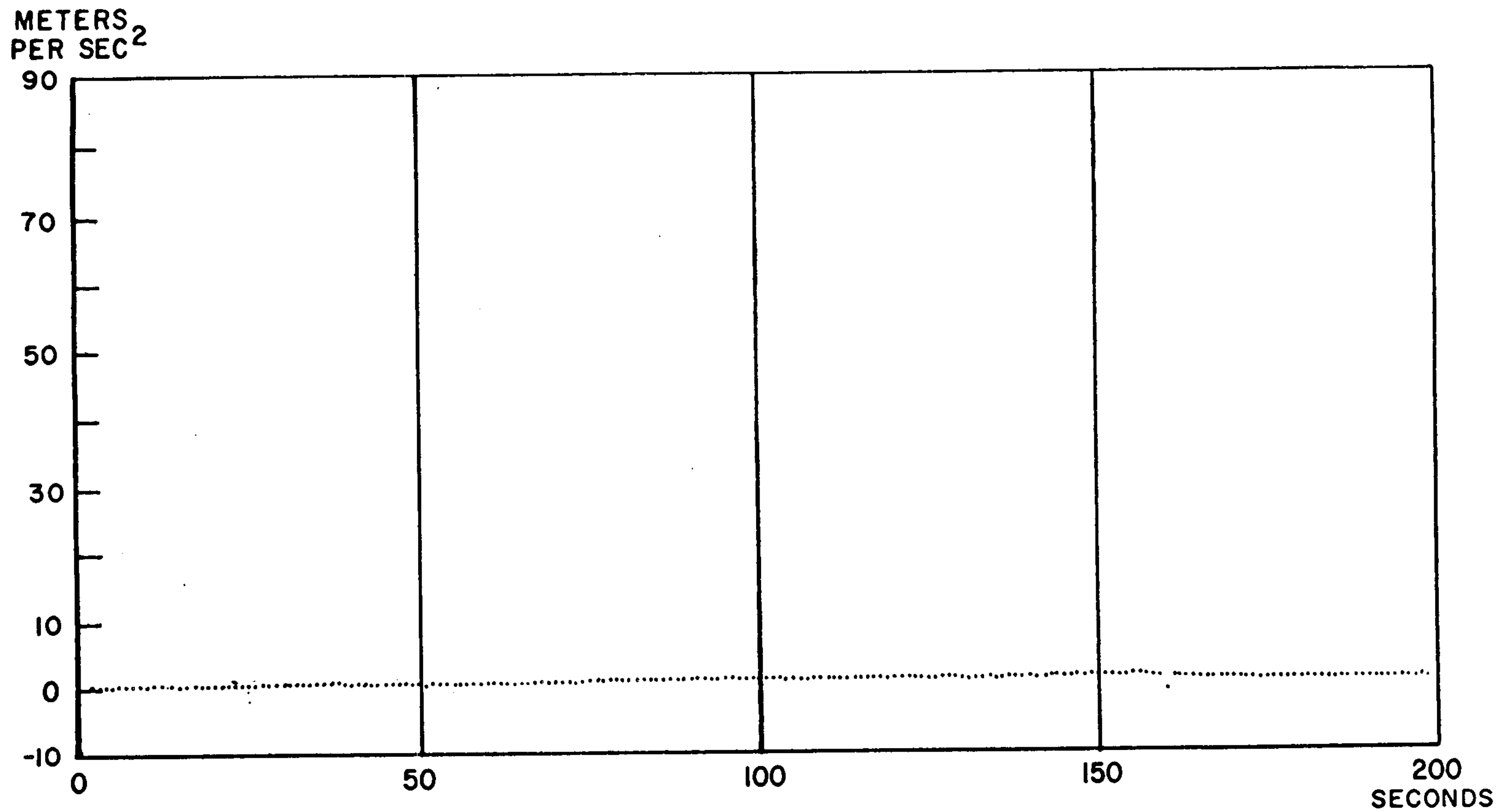

FIG 16 SMOOTH Z ACCELERATION UDOP (0 TO 200 SECONDS)

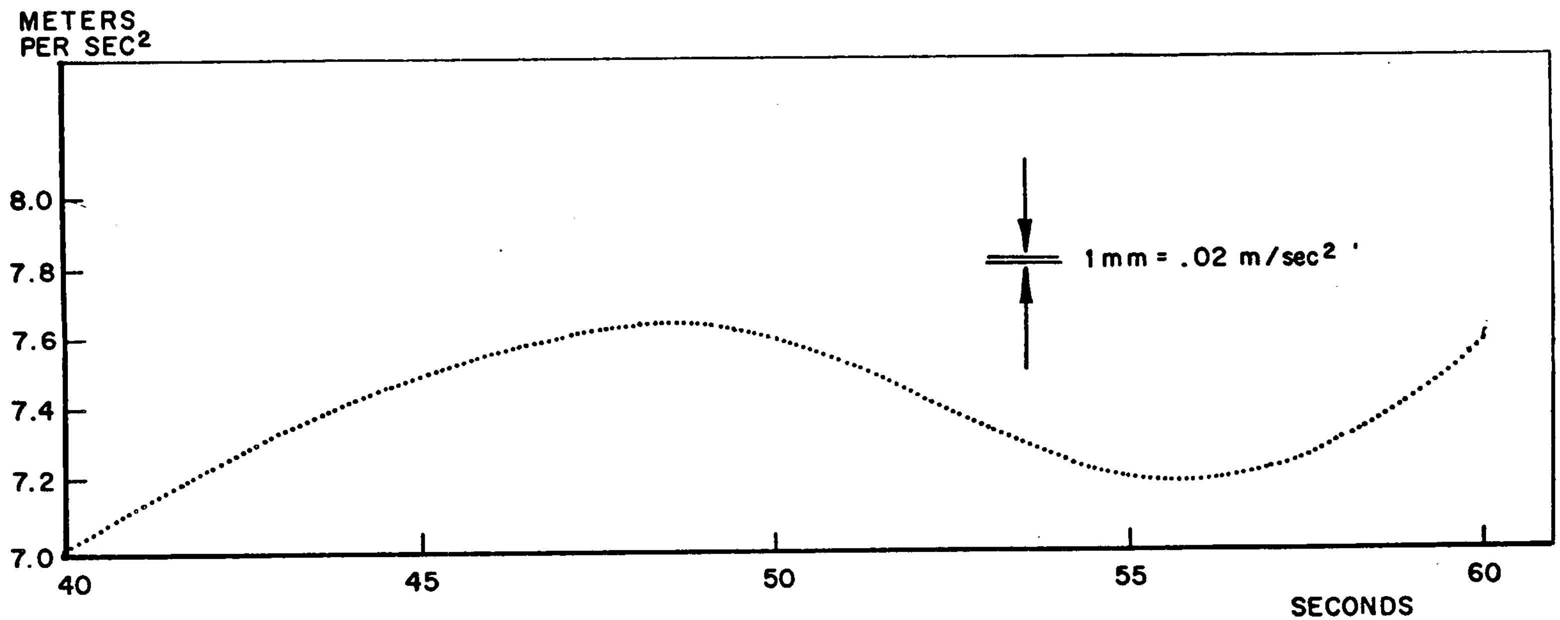

FIG 17. SMOOTH Y ACCELERATION UDOP (40 to 60 sec)

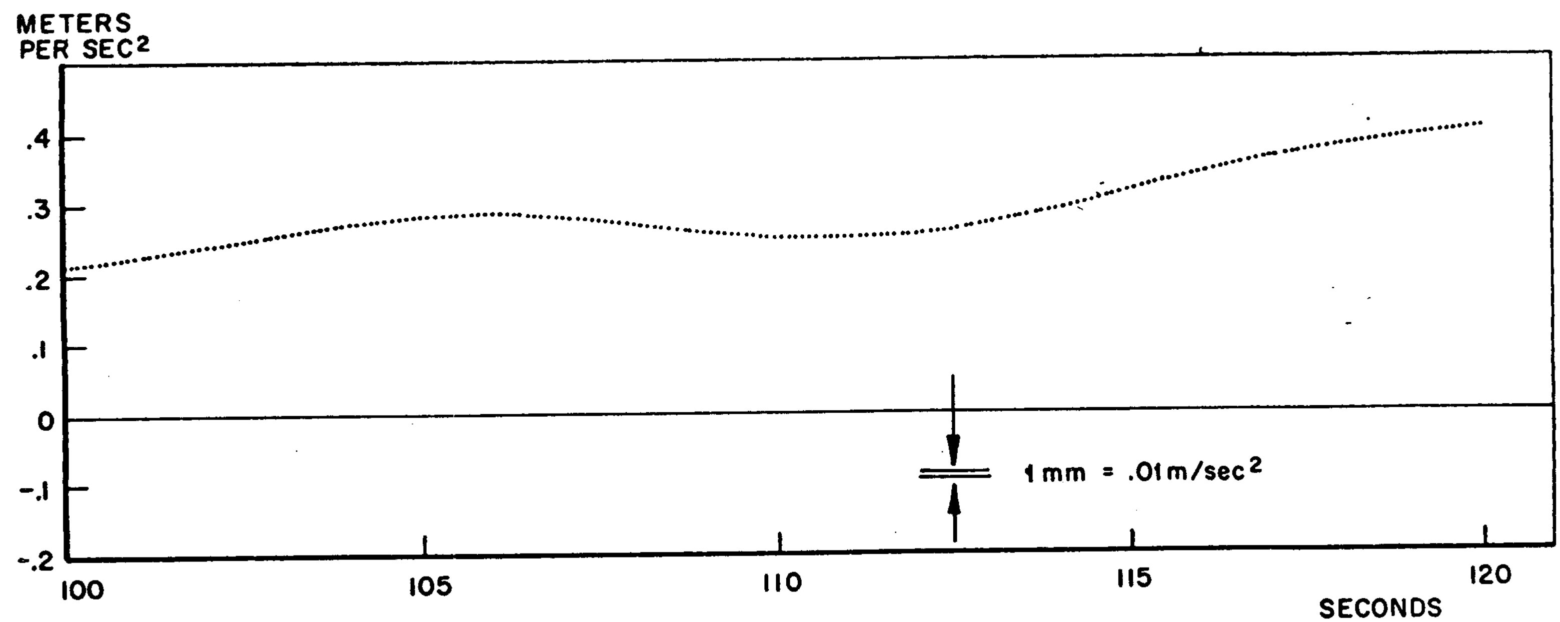

FIG 18. SMOOTH Z ACCELERATION UDOP (100 to 120 sec)

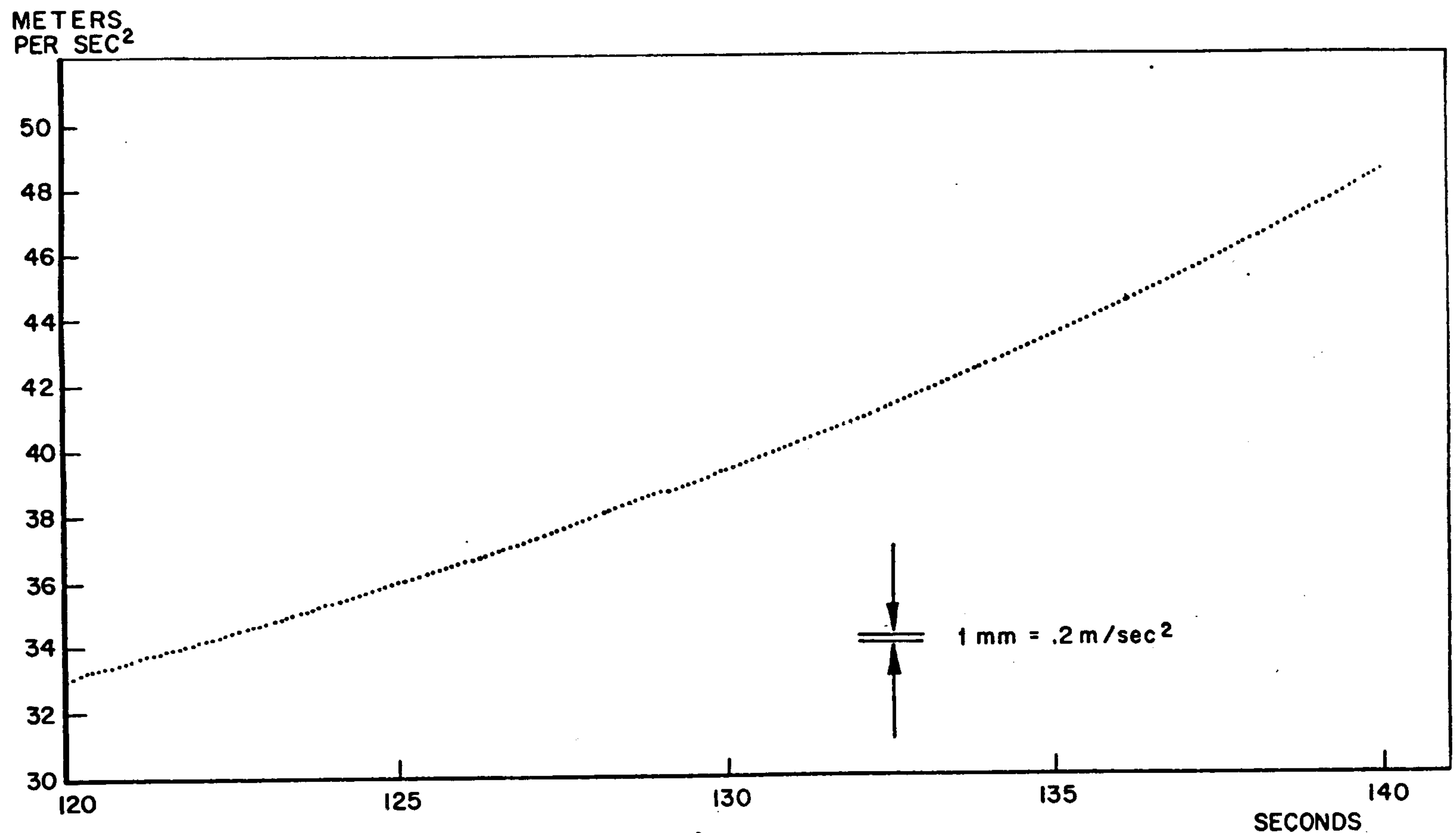

FIG 19. SMOOTH X ACCELERATION UDOP (120 to 140 sec)

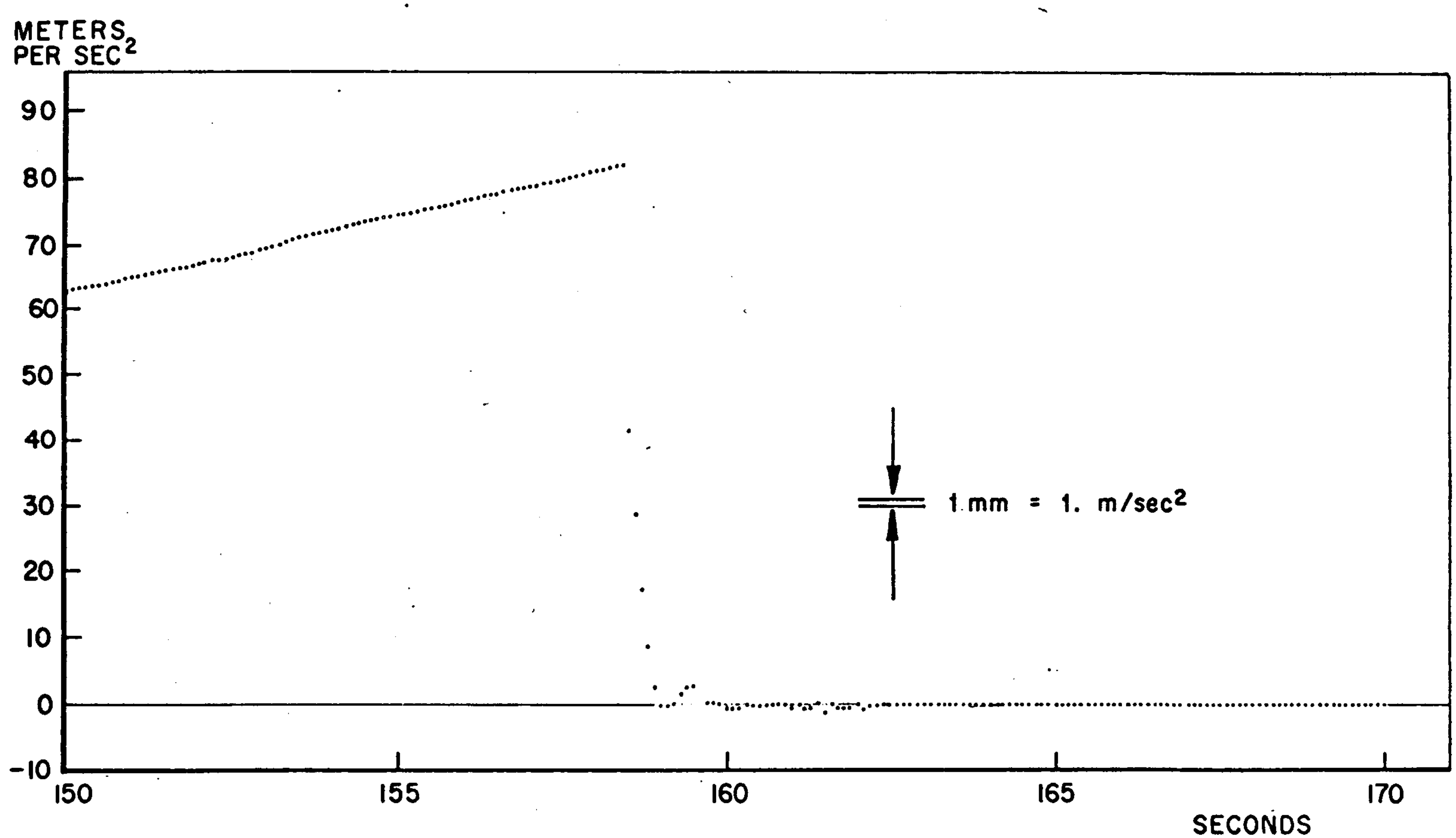

FIG 20. SMOOTH X ACCELERATION UDOP (150 to 170 sec)

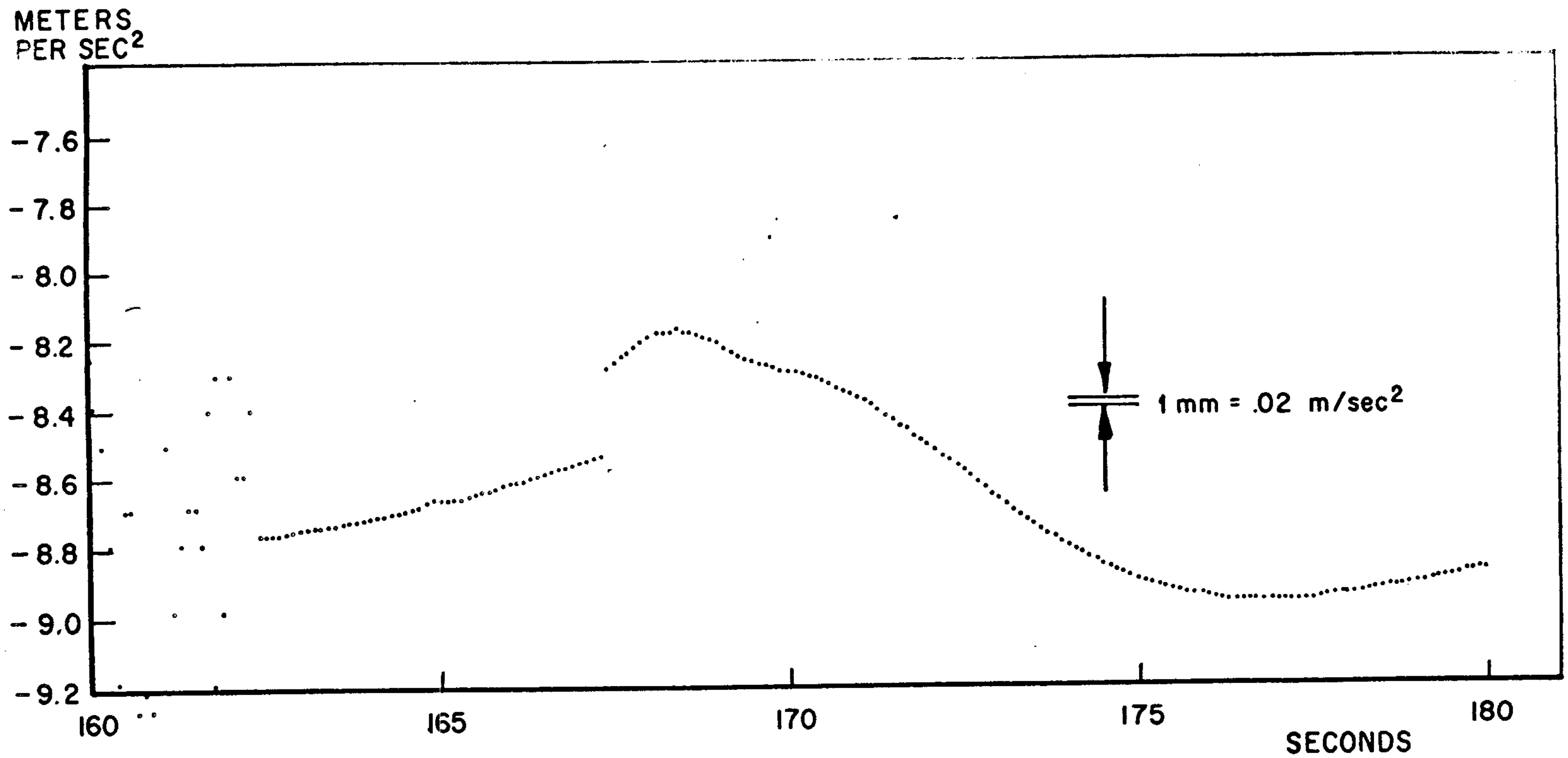

FIG 21. SMOOTH Y ACCELERATION UDOP (160 to 180 sec)

[redacted]

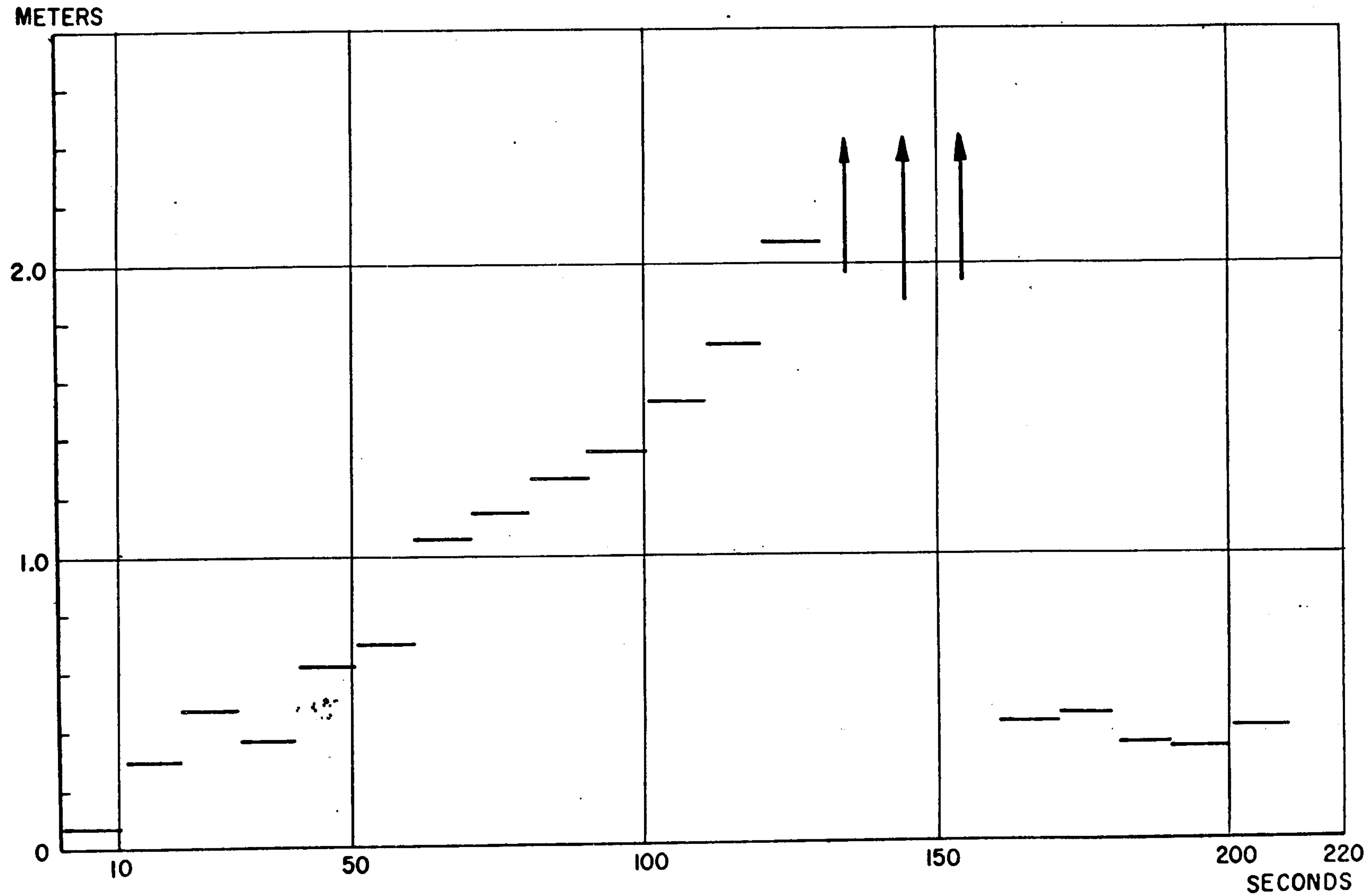

FIG. 22 STANDARD DEVIATION OF SMOOTH X POSITIONS UDOP

[redacted]

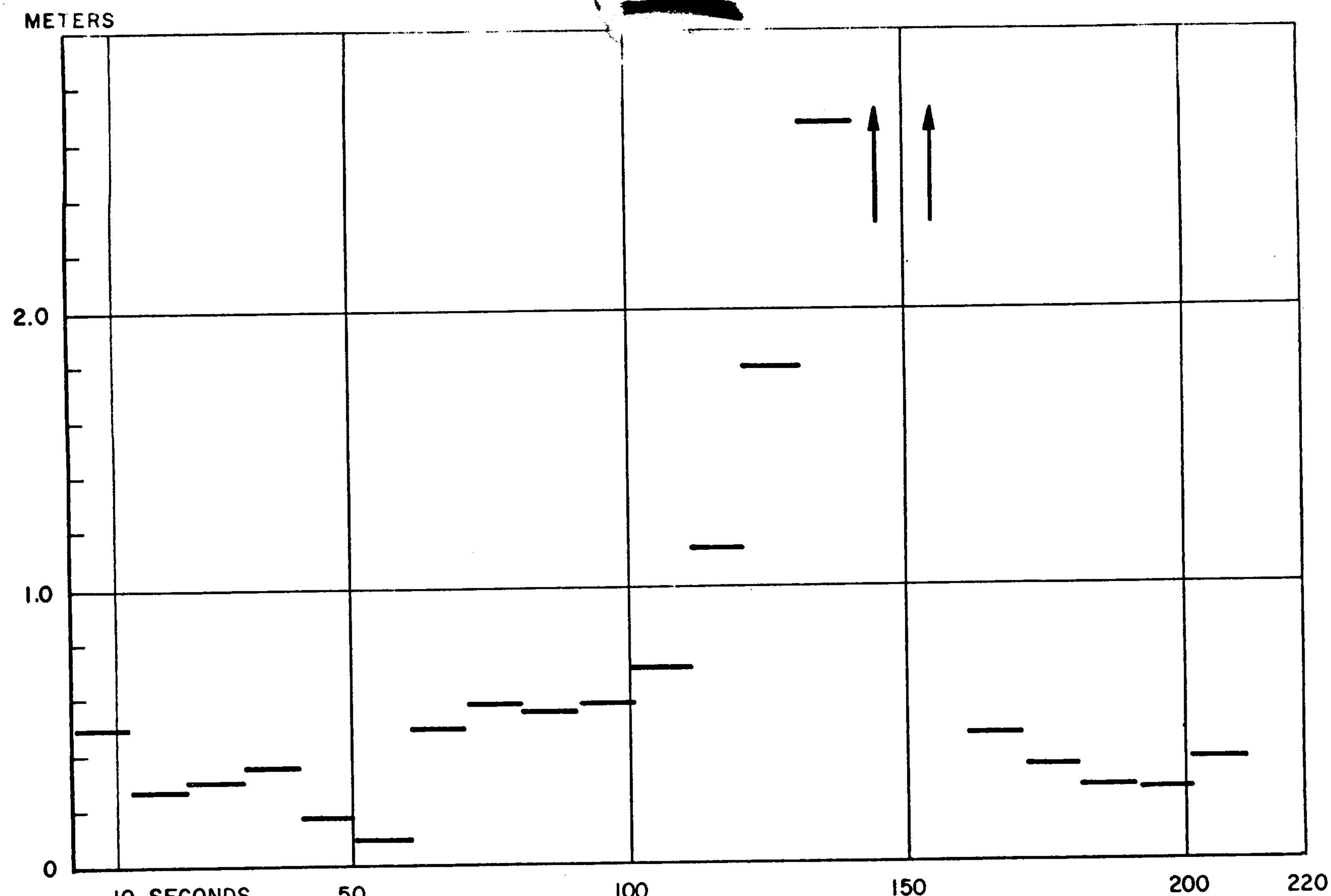

FIG 23 STANDARD DEVIATION OF SMOOTH Y POSITIONS UDOP

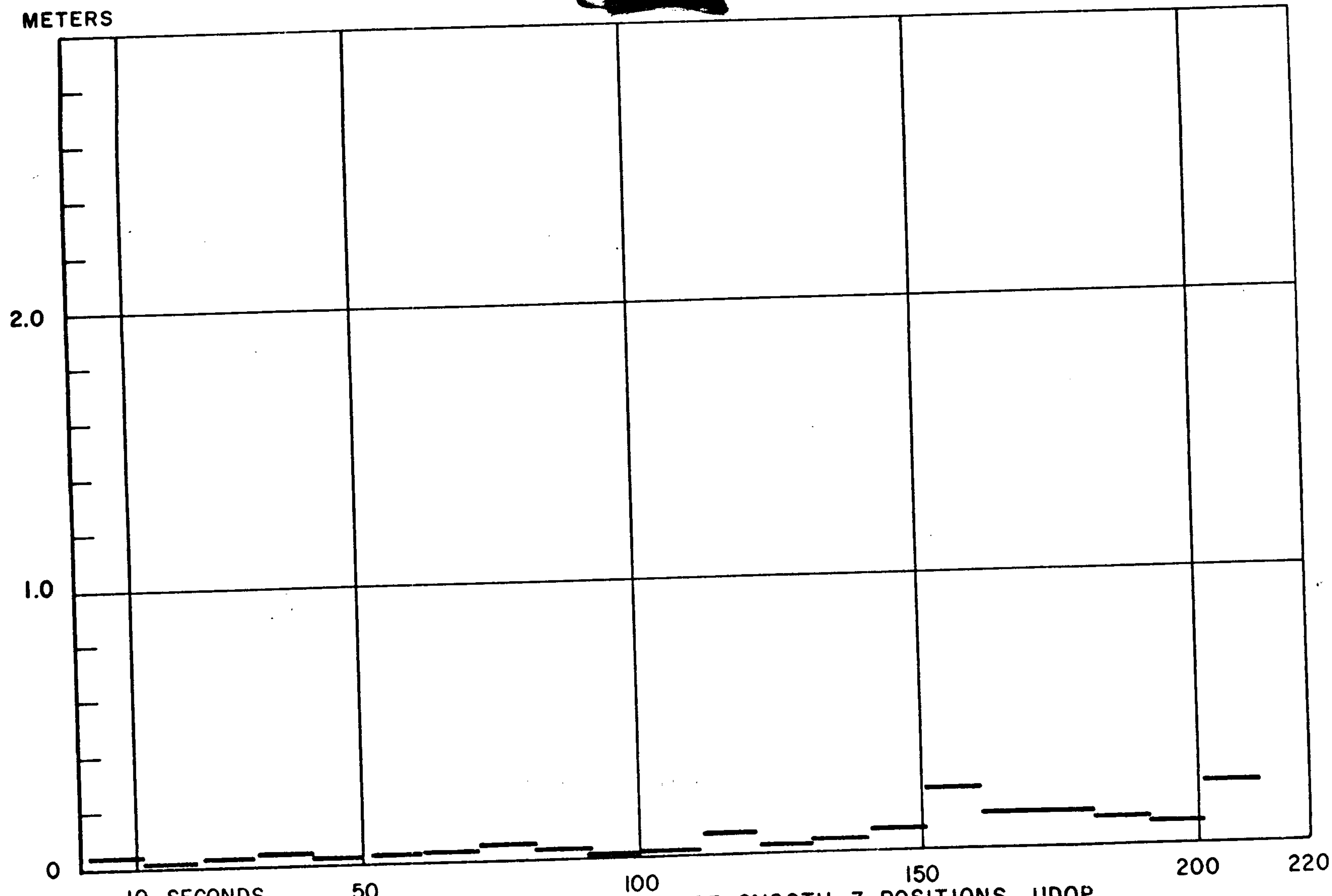

FIG 24 STANDARD DEVIATION OF SMOOTH Z POSITIONS UDOP

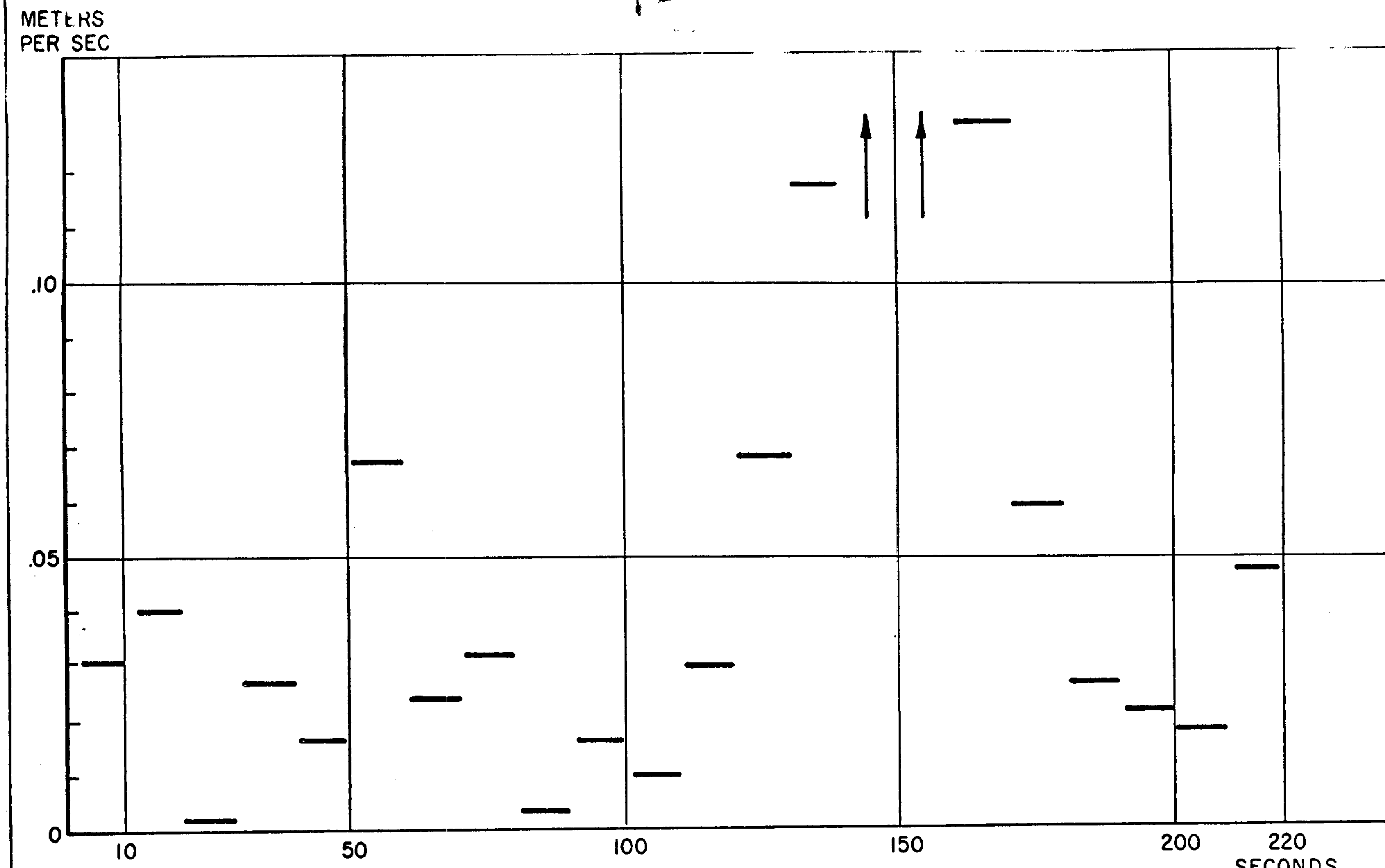

FIG 25 STANDARD DEVIATION OF SMOOTH X VELOCITIES UDOP

FIG 26 STANDARD DEVIATION OF SMOOTH Y VELOCITIES UDOP

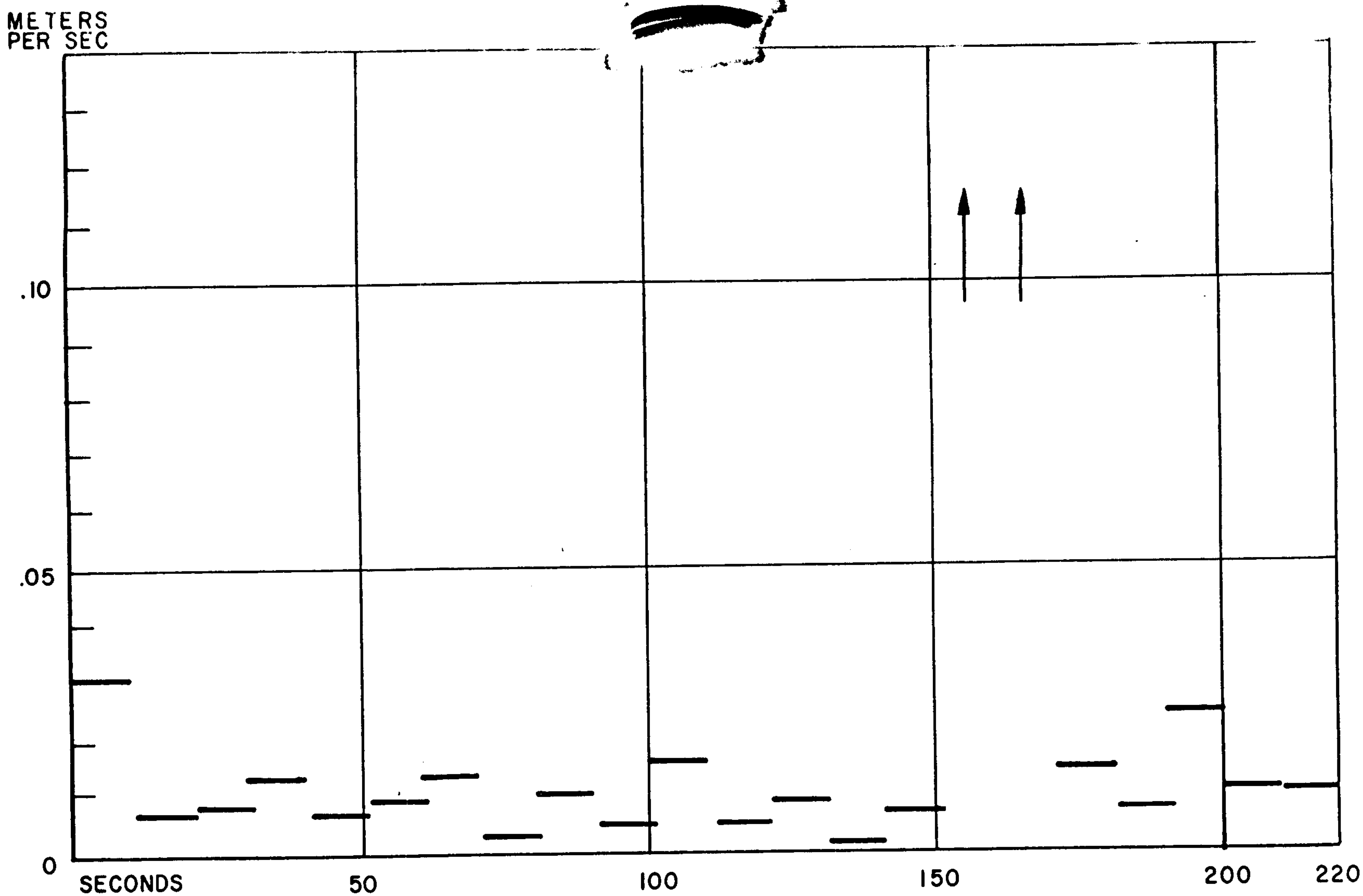

FIG 27 STANDARD DEVIATION OF SMOOTH Z VELOCITIES UDOP

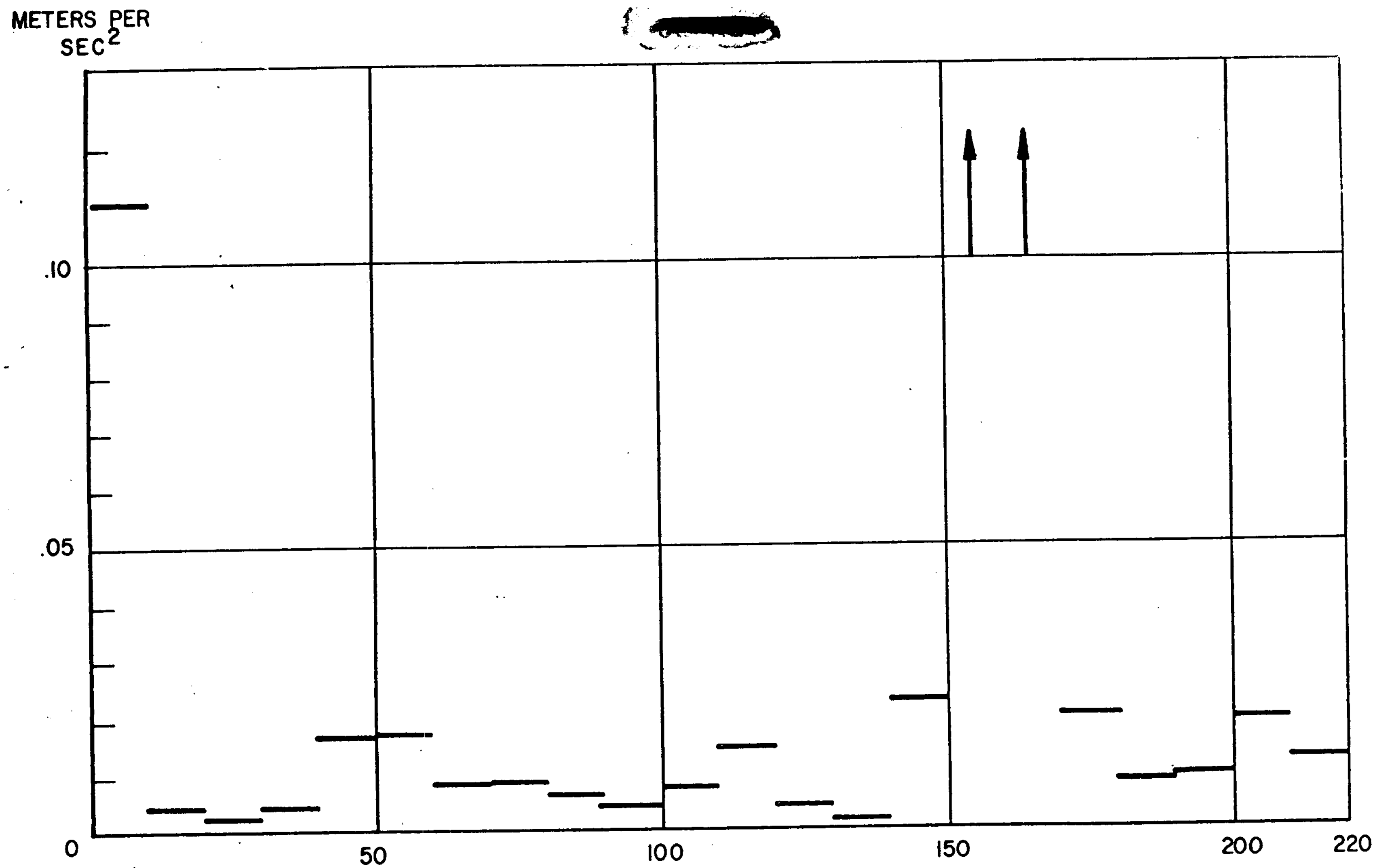

FIG 28 STANDARD DEVIATION OF SMOOTH X ACCELERATIONS UDOP

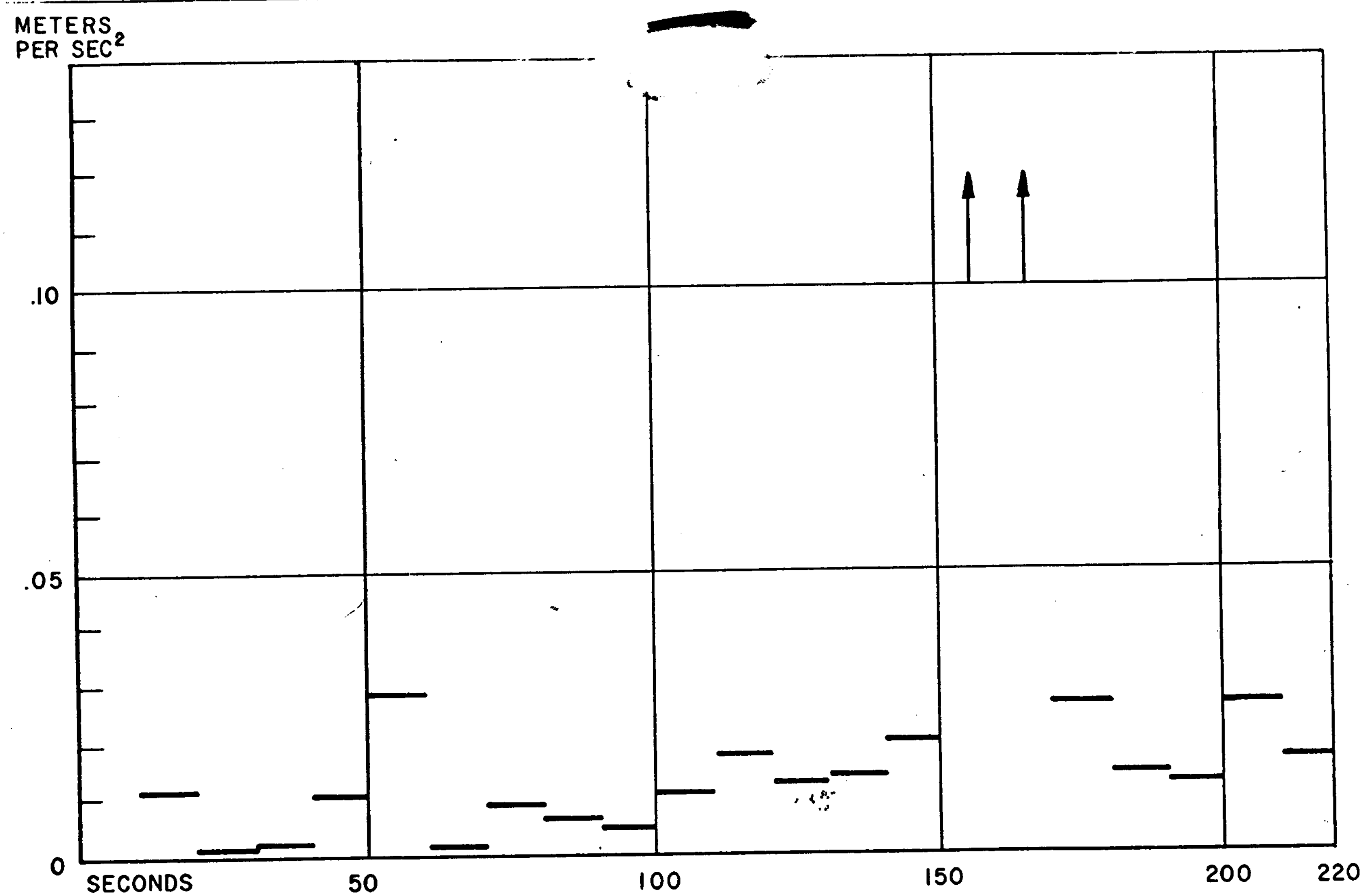

FIG 29 STANDARD DEVIATION OF SMOOTH Y ACCELERATION UDOP

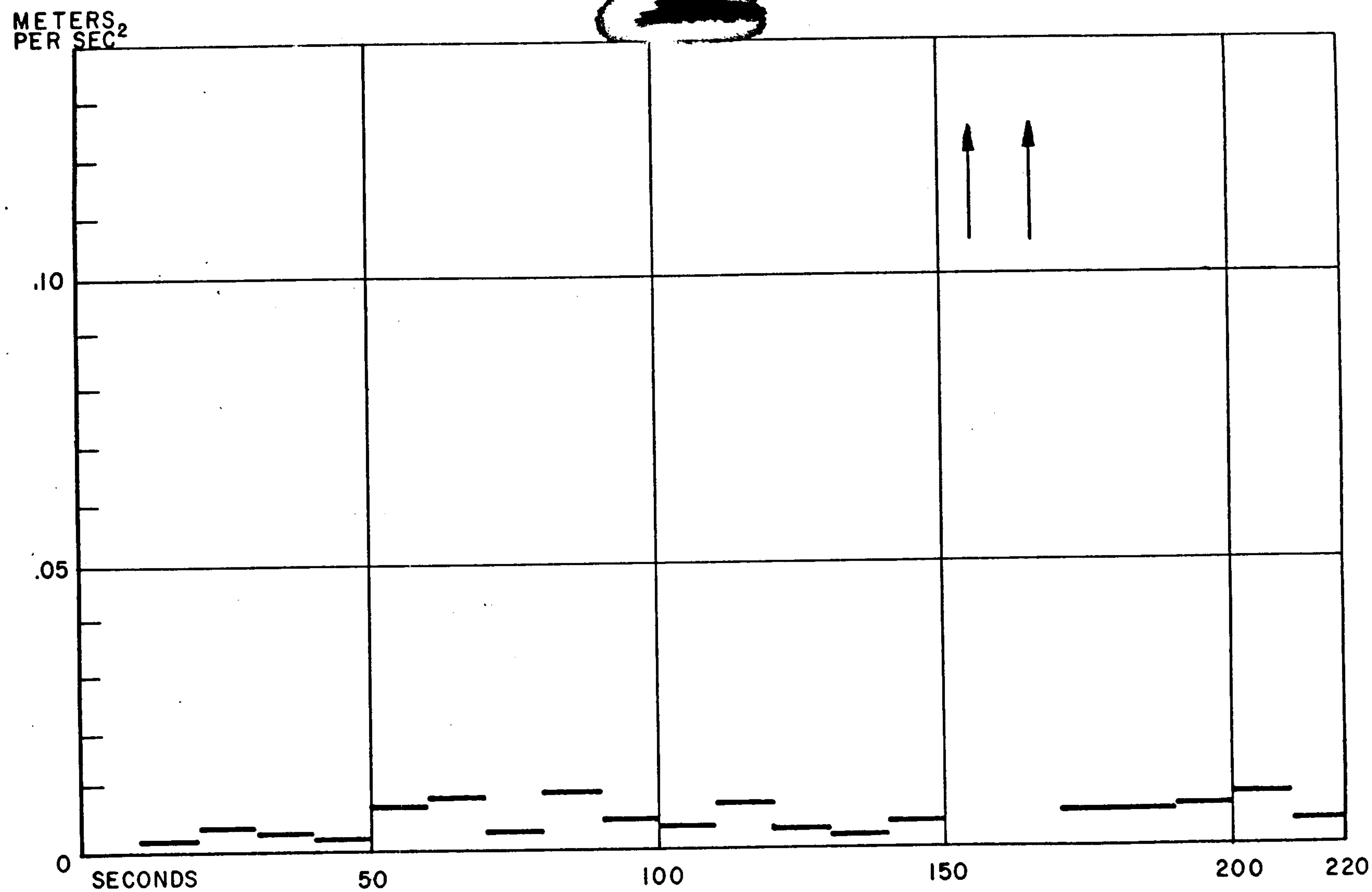

FIG 30 STANDARD DEVIATION OF SMOOTH Z ACCELERATIONS UDOP

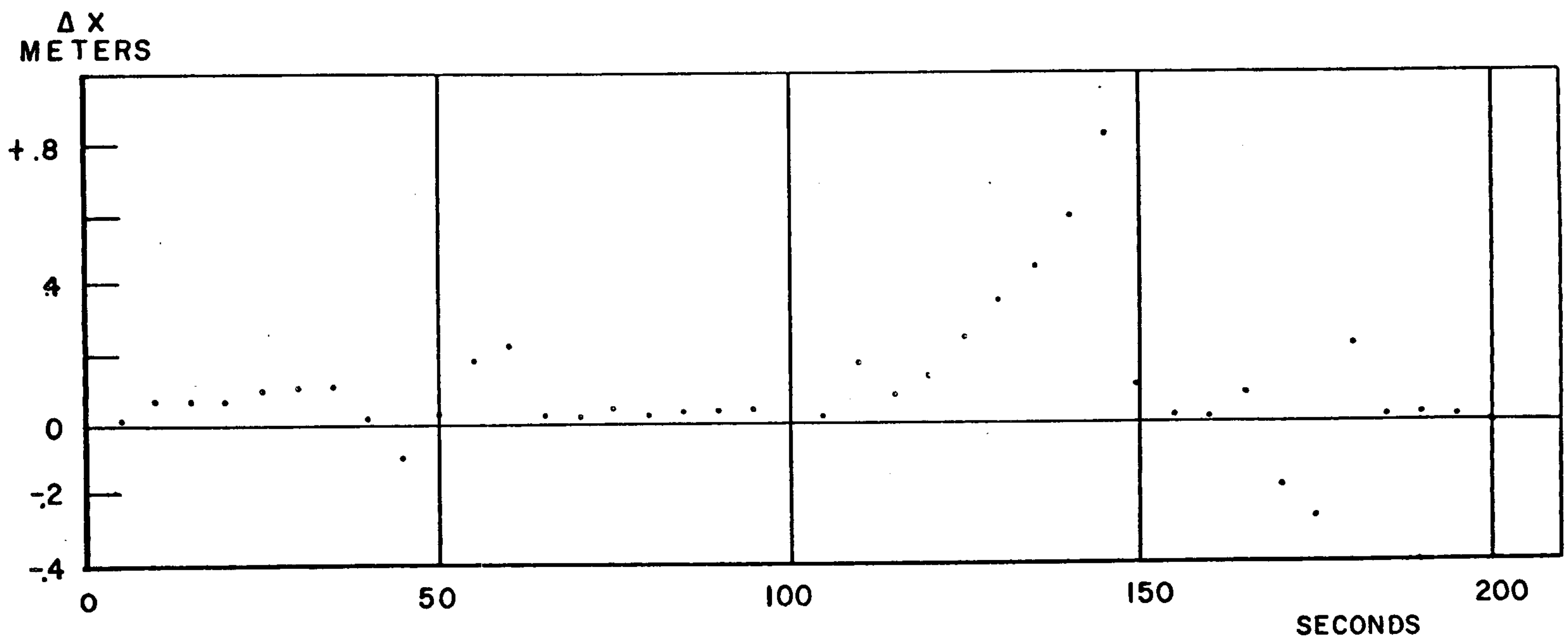

FIG 31 X POSITION ERROR DUE TO SMOOTHING

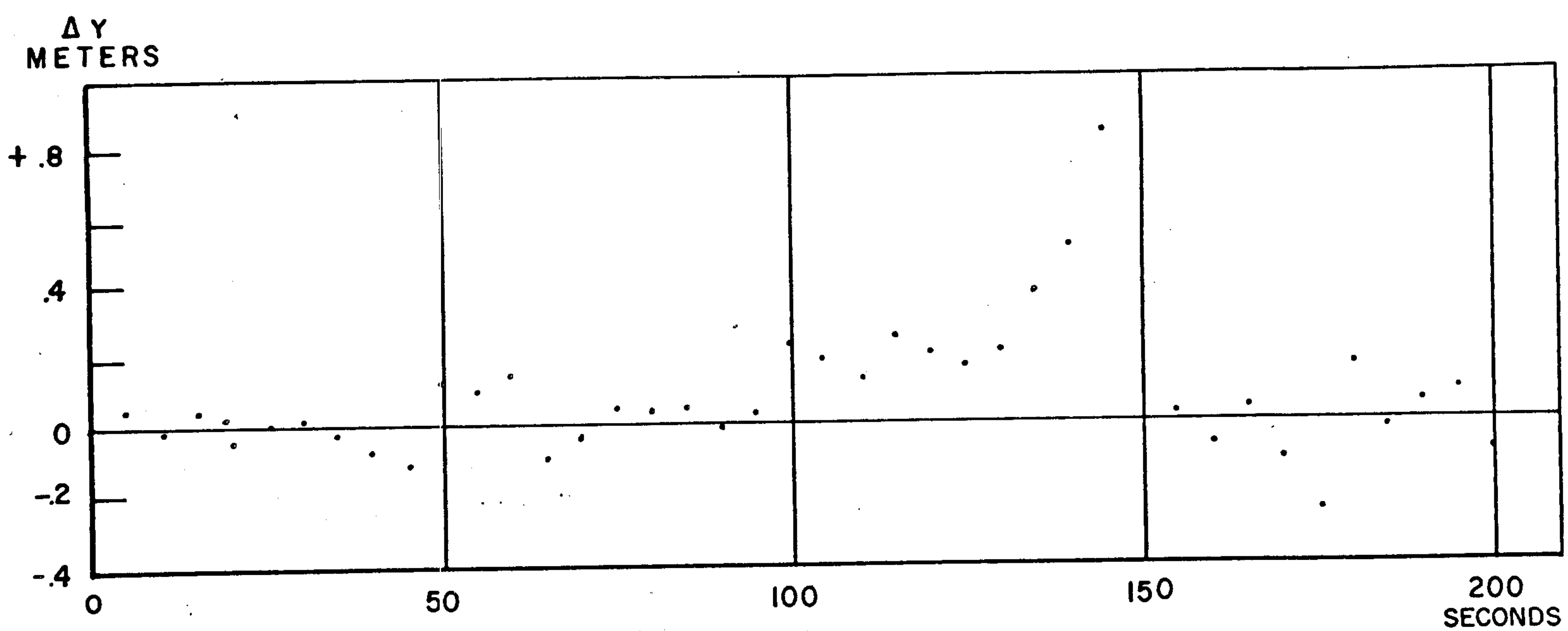

FIG 32 Y POSITION ERROR DUE TO SMOOTHING

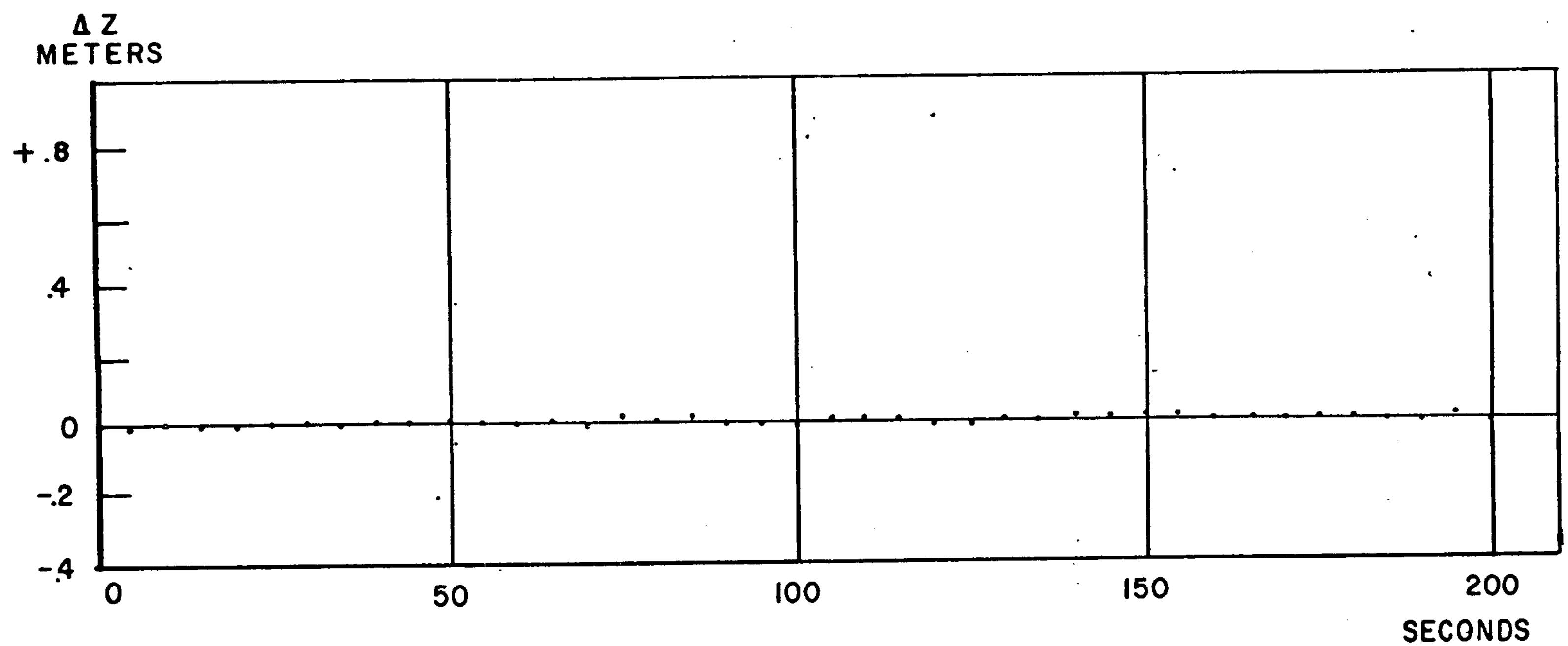

FIG 33 Z POSITION ERROR DUE TO SMOOTHING

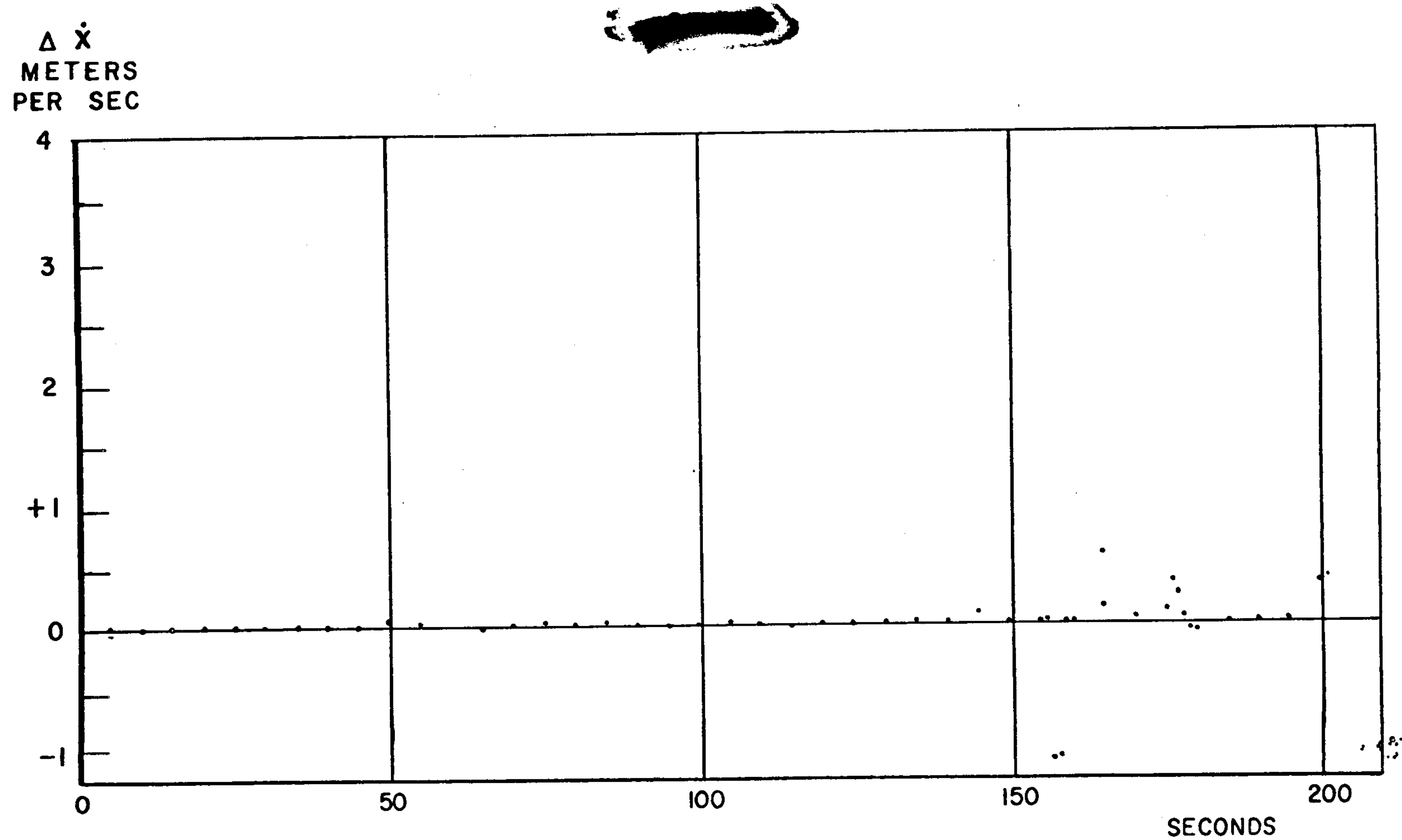

FIG 34 X VELOCITY ERROR DUE TO SMOOTHING

[redacted]

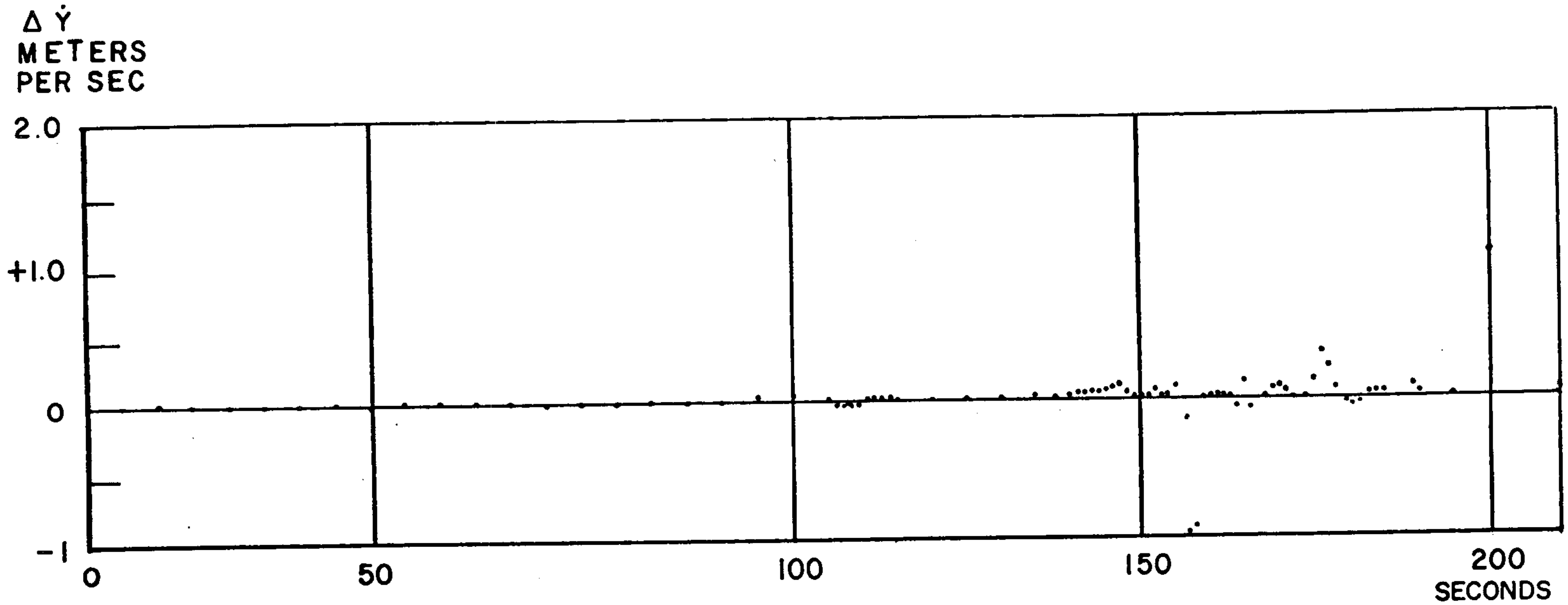

FIG 35 Y VELOCITY ERROR DUE TO SMOOTHING

[redacted]

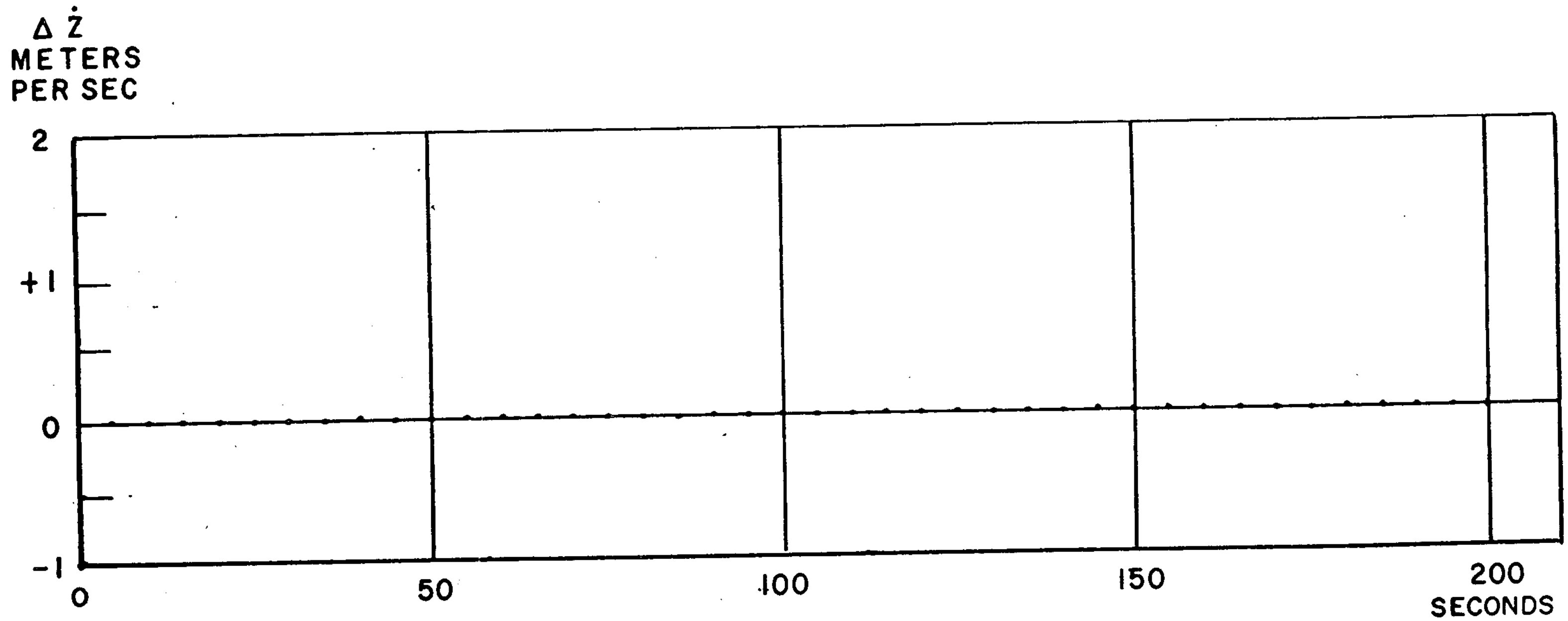

FIG 36 Z VELOCITY ERROR DUE TO SMOOTHING

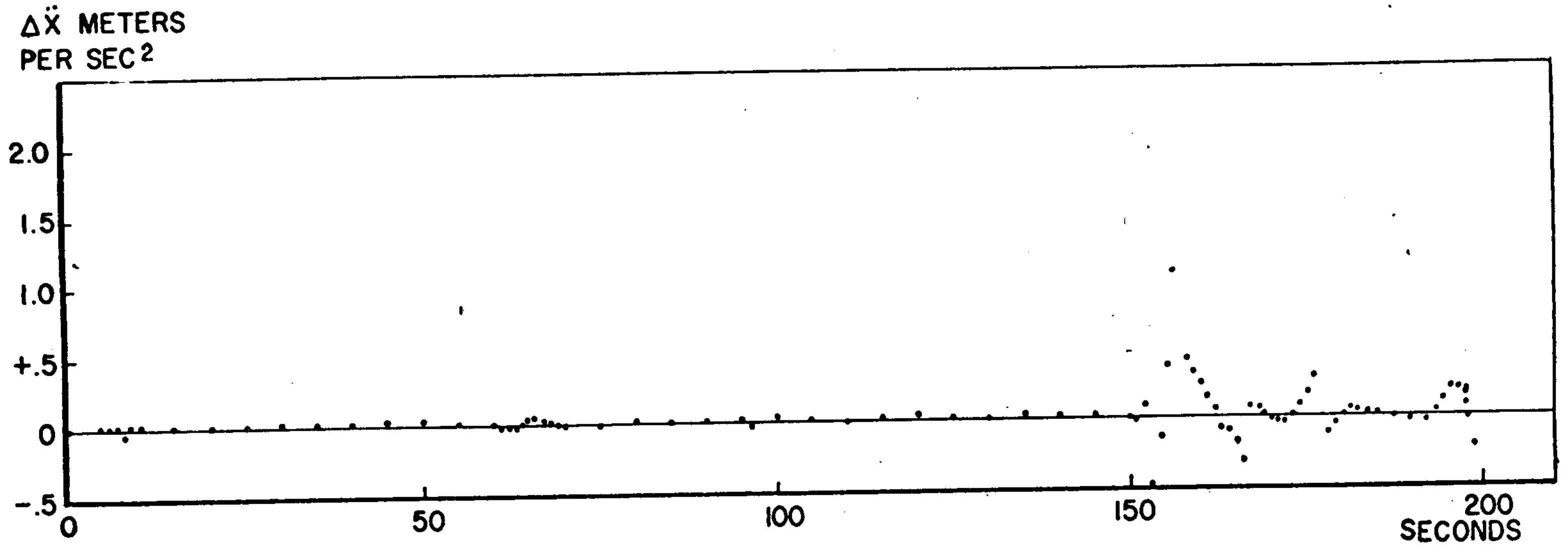

FIG. 37 X ACCELERATION ERROR DUE TO SMOOTHING

[redacted]

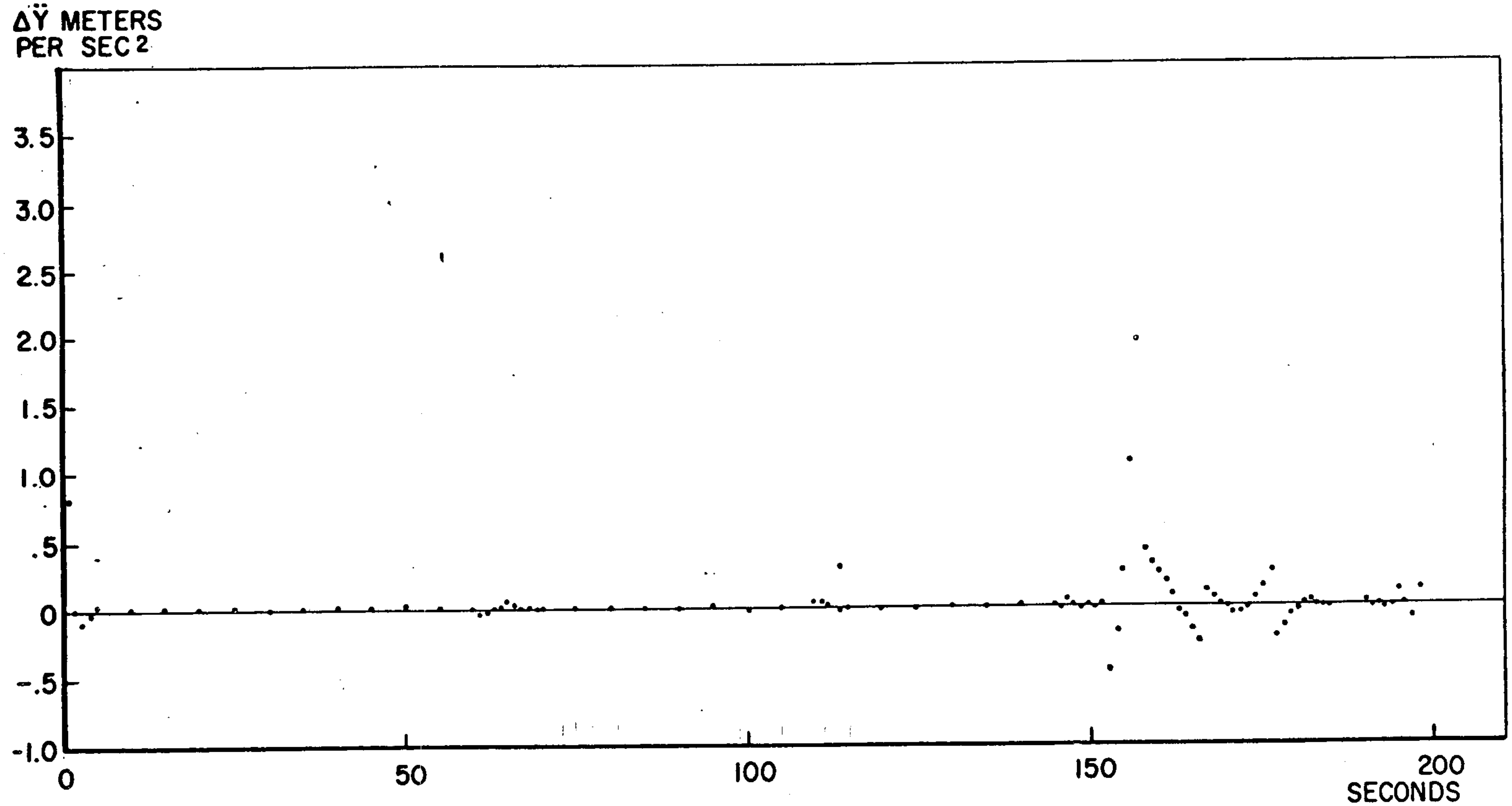

FIG. 38 Y ACCELERATION ERROR DUE TO SMOOTHING

[redacted]

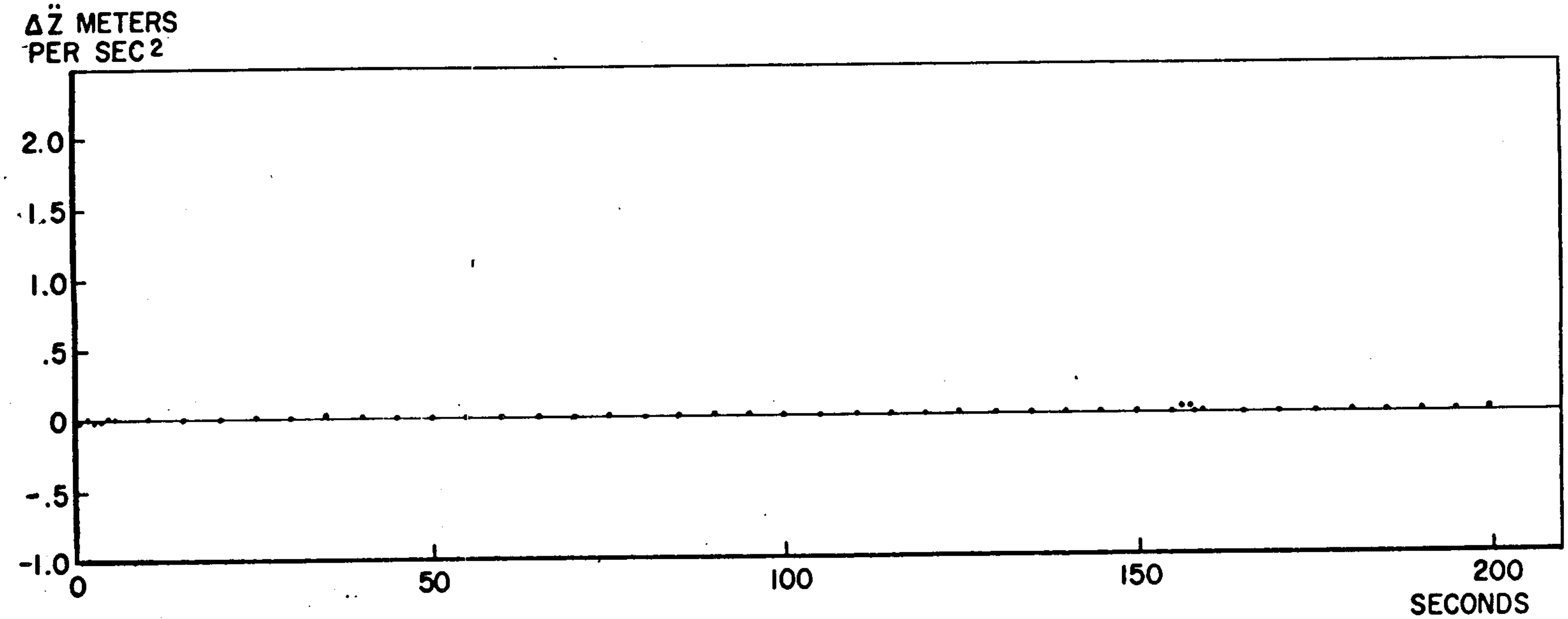

FIG. 39 Z ACCELERATION ERROR DUE TO SMOOTHING

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URRENTLY USED SMO ING AND DIFFEREN ATION
PROCEDURES FOR OBTAINING VELOCITIE AND
ACCELERATIONS AND THE R EFFECT ON DISPE ION (U)

By Philip N An son & Roger A. Ma owan

Classification Changed
ASSIFIED

Aut Security Classification Officer
By

L AERONAUTICS AND SPACE ADMINIST

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July 21, 1960 MM-M-COMP-1-60

CURRENTLY USED SMOOTHING AND DIFFERENTIATION PROCEDURES FOR OBTAINING VELOCITIES AND ACCELERATIONS AND THEIR EFFECT ON DISPERSION (U)

By

Philip N. Anderson and Roger A. MacGowan

This report contains 54 ~~classified~~ UNCLASSIFIED pages.

DATA REDUCTION BRANCH
COMPUTATION DIVISION

Arranged and Processed
by
PUBLICATIONS ENGINEERING SECTION
SPACE SYSTEMS INFORMATION BRANCH

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(U) ABSTRACT

The reasons for the sele:tion of the smoothing and dif-
f entiation formulas, which are currently used in calculation
o smooth missile positions, velocities and accelerations, are
s died. The formulas are described in detail and thei fect
i illustrated. Approximate values of the noise level in the
s ooth data are provided and the magnitude of systematic errors
d to these procedures is estimated.

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(U) LIST OF ILLUSTRATIONS

(U) LIST OF ILLUSTRATIONS (Continued)

.ST OF SYMBOLS

$_n$	Unsmoothed position X, Y, or Z. Subscript refer particular point.
U_L	Unsmoothed position X Y, or Z at last point.
$_n$	Smoothed position X, Y, or Z
$\bar{U}_n$	Unsmoothed velocity X, Y, or Z calculated from smoothed positions.
$_n$	Unsmoothed acceleration X, Y, or Z calculated from smoothed positions.
$_n$	Smoothed velocity X, Y, or Z calculated from smoothed positions.
$_n$	Smoothed acceleration X, Y, or Z calculated from smoothed positions.
t	Time interval of input data.
T_{co}	Time of chamber pressure drop following cutoff signal.
T_{CPL}	Time of chamber pressure level-off following T_{co}.
T_L	Time of last point in input data.
T_{to}	Missile liftoff time.
T_o	Time of first point in input data.

DECL[redacted]IED

EC' I. (S) INTRODUCTIO

the analysis of missile test flights, velocicies and acceler- ti are the bases of many other calculations One method of determin- ng ocities and accelerations is by numerical differentiation of os ion data. The position data may be obtained from ne of several ypes of instrumentation. The data contain random errors of observat and reduction as well as systematic errors. It is usually necessary mooth the data to obtain realistic numerical derivatives. The numeri smoothing and differentiation procedures have undergone considerable vol ionary change as a result of experience with varied instrumentat iss e ystems, and flight paths. The complexity of the procedures inc greatly. Questions have frequently arisen concerning the pres nt smoothing and differentiation procedures and the reaso for sir these procedures. This report provides some answers by giving some insight into the general problem of smoothing and differe atio nd by description of the currently used procedures.

n analyzing smoothing and differentiation procedures it des o ave some means of estimating the dispar ior of noise in position el ties, and accelerations A method has been devised for doing nd described briefly. A method is also descri and applied f det ining the systematic errors introduced by the ing d dif ntiation procedures.

SEC N II (S) SMOOTHING AND DIFFERENTIATION PROCEDURES CURRENTLY IN USE

Development

The smoothing procedures now in use in the Data Redu n gen ly use moving arc smoothing formulas. In this operation a is d to an arbitrary number of points which are usually s al at ed time interval and represent a segment of a time ser s. r points, usually the central point, is adjusted to con ex to e fitted curve. Then the curve fit formula is shifted along tim eries so that one new point is added to the set and one old po at e other end of the series is removed. The fitting and adjustmer procedure is then reapplied to the new set, leading to the adjustmen a point adjacent to the previously adjusted point. This procedure m be continued over a major portion of a time series. This point-by-poi moving arc smoothing reduces the discontinuities due to end effects to a minimum by distributing them among all the intervals.

he early smoothing procedures employed involved unweighted poly nomia approximation by least squares and orthogonal polynomial formul Later it was found that the smoothing formulas derived by L. S. Dederick (Ref 1) were convenient and gave superior results. The goal of a smooth- ing mula is to increase the smoothness of the data without excessi

[redacted]

increasing the adjustments necessary to achieve this smoothness. The smoothness and adjustments may be measured in terms of the magnitude of the n^{th} order differences and the magnitude of the residuals

The velocities and accelerations calculated by numerical differ- iation frequently showed oscillations of considerable amplitude nation or reduction of these oscillations, which were considered lis c, was required. The amplitude of these oscillations increased y with an increasing degree of the smoothing formula. Thus it was able to use as low a degree as possible without causing gross rtion of the original data. It was found that a degree lower than d could not be used with the point spreads that were being considered d degree Dederick smoothing formulas of increasing point spread w pplied to actual data. In that way a high degree of local s ness could be achieved while the data still contained very distinct ations of considerable amplitude. It was apparent that the c llations could be reduced by increasing the point spread of the d degree smoothing so as to encompass several oscillations. Thus orm would not be able to follow the individual oscillations and therefore reduce their amplitudes. Our smoothing formula was ded to cover a 20-second time interval in order to accomplish this tio in the oscillations. One-tenth of a second time steps means a point smoothing formula. This large number of points would the calculation time on a machine appreciable and the build-up of d-off errors might be appreciable also. The difficulty was a viated by using a 101 point, second degree smoothing formula which every second point in the sequence. A further improvement in the smoothness of the velocities and accelerations was achieved by g a second pass smoothing of forty-one points and second degree

This smoothing procedure has the disadvantage of not being able to ve ny physical fluctuation having a period and amplitude similar le than that of the oscillations. The characteristic Mach one rbance is of sufficient period and amplitude to remain distinct er, the characteristic engine cutoff pattern would be grossly rted by this smoothing procedure. In order to preserve the cte stic engine cutoff pattern, the point spread of the smoothing cre ed in steps as the time of cutoff is approached. After cutoff the point spread is increased in steps back to that of the general a. Although this permits the preservation of the general cte istic pattern it leaves both noise and oscillations in the data e v inity of cutoff. Smoother values of accelerations are able for use in other calculations. Therefore a second degree nomial is fitted to the ten seconds of acceleration data immediately ding cutoff. This polynomial is evaluated to get smooth erations for the five seconds immediately preceding cutoff. er cond degree polynomial is fitted to the ten seconds of eration data immediately following the chamber pressure level-off owing cutoff. This polynomial is evaluated to get smooth accelerations the five seconds immediately following chamber pressure level-off

Special procedures are also used for smoothing and differentiation at the beginning and at the end of the time series. These involve the use shorter point spreads and asymmetric formulas.

A general purpose smoothing and differentiation program utilizing Dede's coefficients was developed in the Test Data Processing Section. This program was used in some of our studies. It was possible to select any point spread up through twenty-five and any degree up through four. A number of programs utilizing higher point spreads were prepared by Walton L. Whigham of the Test Data Processing Section for use in the studies.

Obviously the procedures could be greatly improved if the oscillations could be kept from developing. It has been discovered that some contribution to the oscillations may be due to roundoff exceeding the relative accuracy of the data. This phenomenon has been studied and reported (Ref 2). It may be possible to eliminate this source of oscillations. It has also been established that some contribution to the oscillations is due to the smoothing of random noise. This phenomenon has also been studied and reported (Ref. 3). This latter oscillation source cannot be easily eliminated since it is due only to the randomness of the noise and the sampling rate. Other sources of oscillations in the various types of tracking instrumentation also exist.

2. Description

The present smoothing and differentiation procedures are programmed for the IBM No. 709. The input to the program is trajectory position data calculated at a fixed time interval. The program consists of two main parts In the first part the position data are smoothed, and first and second derivatives are calculated at each time step using these smoothed positions. In the second part of the program the calculated velocities and accelerations are smoothed and a second degree curve fit is used to obtain smooth accelerations near cutoff time.

a. Initial equations of the first part

$$\bar{U}_0 = \dot{\bar{U}}_0 = \ddot{\bar{U}}_0 = 0 \qquad \text{when } t_0 \leq t_{to}$$

$$\bar{U}_0 = \frac{1}{5}\,(3U_0 + 2\bar{U}_1 + \bar{U}_2 - \bar{U}_4) \qquad \text{when } t_0 > t_{to} \tag{2}$$

$$\dot{\bar{U}}_0 = \dot{\bar{U}}_1 - (\dot{\bar{U}}_2 - \dot{\bar{U}}_1) \qquad \text{when } t_0 > t_{to}$$

$$\ddot{\bar{U}}_0 = \ddot{\bar{U}}_1 - (\ddot{\bar{U}}_2 - \ddot{\bar{U}}_1) \qquad \text{when } t_0 > t_{to}$$

$$\bar{U}_1 = \frac{1}{5}\,(3U_1 + 2\bar{U}_2 + \bar{U}_3 - \bar{U}_5)$$

$$\dot{\bar{U}}_1 = \frac{\bar{U}_2 - \bar{U}_0}{2\Delta t} \qquad (6)$$

$$\ddot{\bar{U}} \quad \frac{- 2\bar{U}_1 + \bar{U}_0}{\Delta t^2} \qquad (7)$$

$$\bar{U} \quad \sum_{i=-2}^{+2} C_i \, U_{i+2} \qquad (8)$$

re s may be found in Column A of Table 1.

$$\dot{\bar{U}} = \frac{\bar{U}_3 - \bar{U}_1}{2\Delta t} \qquad (9)$$

$$\ddot{\bar{U}} = \frac{\bar{U}_3 - 2\bar{U}_2 + \bar{U}_1}{\Delta t^2} \qquad (10)$$

$$\bar{U} = \sum_{i=-3}^{+3} C_i \, U_{n+i} \qquad \text{when } 3 \leq n \leq 14 \qquad (11)$$

ce s may be found in Column B of Table 1.

$$\dot{\bar{U}} = \frac{\bar{U}_{n+1} - \bar{U}_{n-1}}{2\Delta t} \qquad \text{when } 3 \leq n \leq 14 \qquad (12)$$

$$\ddot{\bar{U}} \quad \frac{\bar{U}_{n+1} - 2\bar{U}_n + \bar{U}_{n-1}}{\Delta t^2} \qquad \text{when } 3 \leq n \leq 14 \qquad (13)$$

$$\bar{U}_n = \sum_{i=-15}^{+15} C_i \, U_{n+i} \qquad \text{when } 15 \leq n \leq 24 \qquad (14)$$

where the C's may be found in Column D of Table 1.

For $\dot{\bar{U}}_n$ and $\ddot{\bar{U}}_n$ when $15 \leq n \leq 24$ see Equations (12) and (13).

$$\bar{U} = \sum_{i=-25}^{+25} C_i \, U_{n+i} \qquad \text{when } 25 \leq n \leq 49 \qquad (15)$$

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where he C's may be found n Column F of Table 1.

; $\dot{\bar{U}}_n$ and $\ddot{\bar{U}}_n$ when $25 \leq n \leq 49$ see Equations (12) and (13).

$$\sum_{i=-50} C_i \, U_{n+i} \qquad \text{when } 50 \leq n \leq 99 \tag{16}$$

re the C's may be found in Column G of Table 1.

For $\dot{\bar{U}}_n$ and $\ddot{\bar{U}}_n$ when $50 \leq n \leq 99$ see Equations (12) and (13).

b. <u>General equations of the first part</u>

$$\bar{U}_n = \sum_{i=-50}^{+50} C_i \, U_{n+2i} \qquad \text{when } 100 \leq n \leq (t_{co} - 101\,\Delta t) \tag{17}$$

ere the C's may be found in Column G of Table 1.

$$\frac{\bar{U}_{n+2} - \bar{U}_{n-2}}{4\Delta t} \tag{18}$$

$$\frac{\bar{U}_{n+2} - 2\bar{U}_n + \bar{U}_{n-2}}{2\Delta t^2} \tag{19}$$

c. <u>Cutoff equations of the first part</u>

r $\bar{U}_n$, $\dot{\bar{U}}_n$, $\ddot{\bar{U}}_n$:

when $t_{co} - 100\,\Delta t \leq n \leq t_{co} - 51\,\Delta t$ see Equations (16), (12) (13

when $t_{co} - 50\,\Delta t \leq n \leq t_{cc}$ 26 Δt see Equations (15), (12) (13).

$$\sum_{i=-10}^{+10} C_i \, U_{n+i} \qquad \text{when } t_{co} - 25\,\Delta t \leq n \leq t_{co} + 25\,\Delta t \tag{20}$$

where the C's may be found in Column C of Table 1.

For $\dot{\bar{U}}_n$, $\ddot{\bar{U}}_n$ when $t_{co} - 25\,\Delta t \leq n \leq t_{co} + 25\,\Delta t$ see Equations (12),

[redacted]

For $\overline{U}_n$, $\dot{\overline{U}}_n$, $\ddot{\overline{U}}_n$:

when $t_{co} + 26\ \Delta t \leq n \leq t_{co} + 50\ \Delta t$ see Equations (15), (12), (13),

wh. $_{co} + 51\ \Delta t \leq n \leq t_{co} + 100\ \Delta t$ see Equations (16), (12), (13)

wh $t_{co} + 101\ \Delta t \leq n \leq t_L - 100\ \Delta t$ see Equations (17), (18), (19).

d. Terminal equations of the first part

For $\overline{U}_n$, $\dot{\overline{U}}_n$, $\ddot{\overline{U}}_n$:

when $t_L - 99\ \Delta t \leq n \leq t_L - 50\ \Delta t$ see Equations (16), (12), (13),

when $t_L - 49\ \Delta t \leq n \leq t_L - 25\ \Delta t$ see Equations (15), (12), (13),

when $t_L - 24\ \Delta t \leq n \leq t_L - 15\ \Delta t$ see Equations (14), (12), (13),

when $t_L - 14\ \Delta t \leq n \leq t_L - 3\ \Delta t$ see Equations (11), (12), (13).

$$\overline{U}_{\ 2} = \sum_{i=-2}^{+2} C_i\ U_{L-2+i} \tag{21}$$

ce s may be found in Column A of Table 1.

$$\dot{\overline{U}}_{L-} = \frac{\overline{U}_{L-1} - \overline{U}_{L-3}}{2\Delta t} \tag{22}$$

$$\ddot{\overline{U}}_{L-2} = \frac{\overline{U}_{L-1} - 2\overline{U}_{L-2} + \overline{U}_{L-3}}{\Delta t^2} \tag{23}$$

$$\overline{U}_{L-1} = \frac{1}{5}\left(3U_{L-1} + 2\overline{U}_{L-2} + \overline{U}_{L-3} - \overline{U}_{L-5}\right) \tag{24}$$

$$\dot{\overline{U}}_{L-1} = \frac{\overline{U}_L - \overline{U}_{L-2}}{2\Delta t} \tag{25}$$

$$\ddot{\overline{U}}_{L-1} = \frac{\overline{U}_L - 2\overline{U}_{L-1} + \overline{U}_{L-2}}{\Delta t^2} \tag{26}$$

$$\bar{U}_L = \frac{1}{5}\left(3U_L + 2\bar{U}_{L-1} + \bar{U}_{L-2} - \bar{U}_{L-4}\right) \quad (27)$$

$$\dot{\bar{U}}_L = \frac{1}{12\Delta t}\left(3\bar{U}_{L-} - 16\bar{U}_{L-3} + 36\bar{U}_{L-2} - 48\bar{U}_{L-1} + 25\bar{U}_L\right) \quad (28)$$

$$\ddot{\bar{U}}_L = \frac{1}{12\Delta t^2}\left(11\bar{U}_{L-4} - 56\bar{U}_{L-3} + 114\bar{U}_{L-2} - 104\bar{U}_{L-} + 35\bar{U}_L\right) \quad (29)$$

Discontinuities exist in the smooth data at the junction points between the different smoothing formulas. These discontinuities result in very wild derivatives at these points. Special treatment was necessary at these changeover points when seven point spread smoothing or greater was used. A maximum of four points of velocity and acceleration were replaced at each junction. These replacements were based on a second degree curve fit through the previous seven points of velocity and acceleration. The method of least squares was used for the curve fit.

e. Junction point replacement equations

$$\dot{U}_n = A_0 + A_1 t + A_2 t^2 \quad (3$$

$$\ddot{U}_n = B_0 + B_1 t + B_2 t^2$$

where A_0, A_1, A_2, B_0, B_1, B_2 are the coefficients obtained by the above mentioned least squares curve fits.

f. General equations of the second part

$$\dot{\bar{U}}_n = \sum_{i=-20}^{+20} C_i \dot{U}_{n+i} \quad \text{when } 20 \le n \le t_{co} - 51\,\Delta t \text{ and} \quad (32)$$
$$\text{when } t_{CPL} + 51\,\Delta t \le n \le t_L - 20\,\Delta t$$

where C's may be found in Column E of Table 1.

$$\ddot{\bar{U}}_n = \sum_{i=-20}^{+20} C_i \ddot{U}_{n+i} \quad \text{when } 20 \le n \le t_{co} - 51\,\Delta t \text{ and}$$
$$\text{when } t_{CPL} + 51\,\Delta t \le n \le t_L - 20\,\Delta t$$

where C's may be found in Column E of Table 1.

g. Cutoff equation of the second part

$$\ddot{\bar{U}}_n = A_0 + A_1 t + A_2 t^2 \quad \text{when } t_{co} - 50\,\Delta t \le n \le t_{co} \quad (34)$$

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This second degree curve fit is obtained by applying the method of least squares to the one hundred points preceding t_{CO}.

$$\ddot{U} = B_0 + [illegible] + B_2 t^2 \qquad \text{when } t_{CPL} \leq n \leq t_{CPL} + 50 \Delta t \qquad (35)$$

This second degree curve fit is obtained by applying the method of least squares to the one hundred points following t_{CPL}.

The actual calculations in the program are carried out in two distinct passes through the data. The smoothing of the positions and the calculation of velocities and accelerations are done in the first pass. The application of the various formulas of the first pass to the particular parts of the trajectory is summarized in Figure 1. Details of the application of the smoothing formulas in the first pass are illustrated in Figures 3 through 5.

The smoothing of velocities and accelerations and the curve fitting of accelerations in the vicinity of cutoff are done in the second pass. The application of the various formulas of the second pass to the particular parts of the trajectory is summarized in Figure 2.

Effects

The significant factor concerning these smoothing and differentiation procedures is their effectiveness in producing smooth and realistic velocity and acceleration data. Figures 6 through 8 show velocities which were calculated by the current smoothing and differentiation program. The data used in the calculations were at a one-tenth second time interval whereas the data used in the graphs were selected at one second time intervals. Figures 9 through 13 show segments of the velocity data on a smaller scale in order to illustrate effectively the local smoothness. These data are at the one-tenth second time interval which was used in the calculations. Figures 14 through 16 show accelerations which were calculated by the current smoothing and differentiation program. The data used in the calculations were at one-tenth second time interval whereas the data used in the graphs were selected at one-second time intervals. Figures 17 through [illegible] show segments of the acceleration data on a smaller scale in order to illustrate effectively the local smoothness. These data are at the one-tenth second time interval which was used in the calculations.

It will be noted that some problem areas remain in these procedures. The discontinuities in accelerations at the end points of the curve fit data preceding and following cutoff represent a difficulty which needs further improvement. The noise and oscillations which remain in the acceleration data for the period of thrust decay represent another problem area. These difficulties are clearly manifested in Figures 20 and 21 and will be eliminated as time permits.

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is sometimes desirable to have a quantitative measure of the spe on of the noise in order to compare the relative merits of iffe nt smoothing techniques. In order to estimate the dispersion noise in our positions, velocities and accelerations a curve tti program was used to fit these smooth data at successive time nter A second degree polynomial was fitted to each ten-second ter and residuals were calculated. The standard deviation of the sio s was calculated for each interval. It was assumed that the econd degree polynomial was capable of following the general trend f the data over most of the ten-second intervals. It was also assumed hat the second-degree polynomial was not capable of following the ise or other minor fluctuations in a ten-second interval. Therefore e standard deviation of the residuals should be a fair estimate of e noise level of the smoothed data. Figures 22 through 30 show these alculated standard deviations for the smoothed positions, velocities nd accelerations. Generally the noise level in smooth UDOP positions s less than 2.0 meters, in smooth UDOP velocities is less than .07 eter per second, and in smooth UDOP accelerations is less than .02 ter per second per second.

e achievement of smoothness is of little value if it is attained the expense of gross distortion of the original data. It would ertainly not be feasible to use smoothing formulas which regularly roduced systematic errors which exceed the noise level of the smoothed data It was therefore desirable to determine the magnitude of systematic rrors produced by our current smoothing and differentiation procedures synthetic trajectory program was used to generate smooth positions, locities and accelerations representative of a typical missile flight ese smooth positions were then used as input to our current smoothing nd differentiation program. These smoothed positions, velocities and ccelerations were then differenced with the smooth positions, velociti nd accelerations generated by the synthetic trajectory. The difference ndic e systematic errors introduced by the smoothing and differenti n rogram. Figures 31 through 39 show these differences. As might be xpected the differences only become appreciable at times of radical hysical change such as main engine cutoff (157.77 seconds), vernier ngine ignition (166.28 seconds), and vernier engine cutoff (176.29 seconds).

ECTION III (S) CONCLUSIONS

t may be concluded that the current smoothing and differentiation procedures are satisfactory for most parts of a typical missile test flight and for typical tracking instrumentation. The exceptions are the times of rapid physical change such as main engine cutoff, vernier engine ignition, and vernier engine cutoff. The attainment of equival accuracy at these times requires additional observation and special treatment. Investigation of these possibilities will proceed as time permits.

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The noise level in the smooth UDOP positions is generally less than 2.0 meters. In the smooth UDOP velocities the noise level is generally less than .07 meter per second and in the smooth UDOP accelerations is generally less than .02 meter per second per second. The systematic errors introduced by the smoothing and differentiation procedures are generally less than the noise levels of the smooth data.

[redacted]

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TAB[illegible]

GENERAL DEDERICK COEFFICIENTS

i	C_i A	B	C	D	E	F	G
-50							-.00002743
-49							-.00010058
-48							-.00022983
-47							-.00041900
-46							-.00066634
-45							-.00096567
-44							-.00130722
-43							-.00167849
-42							-.00206506
-41							-.00245117
-40							-.00282041
-39							-.00315619
-38							-.00344223
-37							-.00366293
-36							-.00380373
-35							-.00385142
-34							-.00379429
-33							-.00362241
-32							-.00332768
-31							-.00290399
-30							-.00234725
-29							-.00165542
-28							-.00082850
-27							.00013152
-26							.00122070
-25						-.00032438	.00243325
-24						-.00108383	.00376162
						-.00223633	.00519666

[redacted]

TABLE

	C_1						
	A	B	C		E	F	
-22						-.00363836	.00[illegible]72771
-21						-.00508775	.00834281
-20					-.00068479	-.00635536	.01002880
-19					-.00217926	-.00721713	.01177154
-18					-.00424676	-.00746965	.01355607
-17					-.00644783	-.00694741	.01[illegible]36680
-16					-.00826372	-.00553220	.01718766
-15				-.00171211	-.00918903	-.00315853	.01900233
-14				-.00500293	-.00879756	.00018451	.02079439
-13				-.00875514	-.00678499	.00445470	.02254753
-12				-.01150094	-.00299216	.00956047	.02424568
-11				-.01186182	.00258781	.01536678	.02587324
-10			-.00557029	-.00882682	.00981543	.02170263	.02741519
-9			-.01345481	-.00189905	.01842084	.02836945	.02885725
-8			-.01761357	.00886228	.02802735	.03515033	.03018606
-7			-.01289565	.02287915	.03818004	.04181938	.03138929
-6			.00311907	.03914878	.04837719	.04815117	.03245574
-5			.02962788	.05637424	.05810267	.05392963	.03337550
-4			.06303804	.07311382	.06685749	.05895644	.03414001
-3		-.05874125	.09795617	.08793353	.07418899	.06305848	.03474215
-2	-.07342657	.05874125	.12842263	.09954972	.07971656	.06609416	.03517629
-1	.29370629	.29370629	.14913596	.10695119	.08315262	.06795860	.03543841
0	.55944055	.41258741	.1564691[illegible]	.10949229	.08431822	.06858732	.03552606
+1	.29370629	.29370629	.14913596	.10695119	.08315262	.06795860	.03543841
+2	-.07342657	.05874125	.12842263	.09954972	.07971656	.06609416	.03517629
+3		-.05874125	.09795617	.08793353	.07418899	.06305848	.03474215
+4			.06303804	.07311382	.06685749	.05895644	.03414001
+5			.02962788	.05637424	.05810267	.05392963	.03337550
+6			.00311907	.03914878	.04837719	.04815117	.03245574
+7			.01289565	.02287915	.03818004	.04181938	.03138929

[redacted]

TABLE 1 (Continued)

i	ψ_i A	B	C	D	E	F	G
+8			-.01761357	.00886228	.02802735	.03515033	.03018[illegible]
+9			-.01345481	-.00189905	.01842084	.02836945	.028857[illegible]
+10			-.00557029	-.00882682	.00981543	.02170263	.027415[illegible]
+11				-.01186182	.00258781	.01536678	.025873[illegible]
+12				-.01150094	-.00299216	.00956047	.02424568
+13				-.00875514	-.00678499	.00445470	.02254753
+14				-.00500293	-.00879756	.00018451	.02079439
+15				-.00171211	-.00918903	-.00315853	.01900233
+16					-.00826372	-.00553220	.01718766
+17					-.00644783	-.00694741	.01536[illegible]
+18					-.00424676	-.00746965	.01355607
+19					-.00217926	-.00721713	.01177154
+20					-.00068479	-.00635536	.01002880
+21						-.00508725	.00834281
+22						-.00363836	.00672771
+23						-.00223633	.00519666
+24						-.00108383	.00376162
+25						-.00032438	.00243325
+26							.00122070
+27							.00013152
+28							-.00082850
+29							-.00165542
+30							-.00234725
+31							-.00290399
+32							-.00332768
+33							-.00362241
+34							-.00379429
+35							-.00385142
+36							-.00380373

[redacted]

i	C_i A	B	C	D	E	F	G
+37							-.00366293
+38							-.00344223
+39							-.00315619
+40							-.00282041
+41							-.00245117
+42							-.00206506
+43							-.00167849
+44							-.00130722
+45							-.00096567
+46							-.00066634
+47							-.00041900
+48							-.00022983
+49							-.00010058
+50							-.00002743

(U) REFERENCES

1. Dederick, L. S., Construction and Selection of Smoothing Formulas. Report No. 863, Ballistic Research Laboratories; Aberdeen Proving Ground, Maryland. 1953

2. Beard, L. Neel and MacGowan, Roger A., Oscillations In Tracking Data Resulting from Roundoff. Report No. DC-TR-1-60, Army Ballistic Missile Agency, Computation Laboratory; Redstone Arsenal, Alabama. 1960

3. Beard, L. Neel and MacGowan, Roger A., Oscillatory Phenomena In Empirical Data Resulting from the Smoothing of Random Noise. Report No. DC-TR-3-60, Army Ballistic Missile Agency, Computation Laboratory; Redstone Arsenal, Alabama. 1960

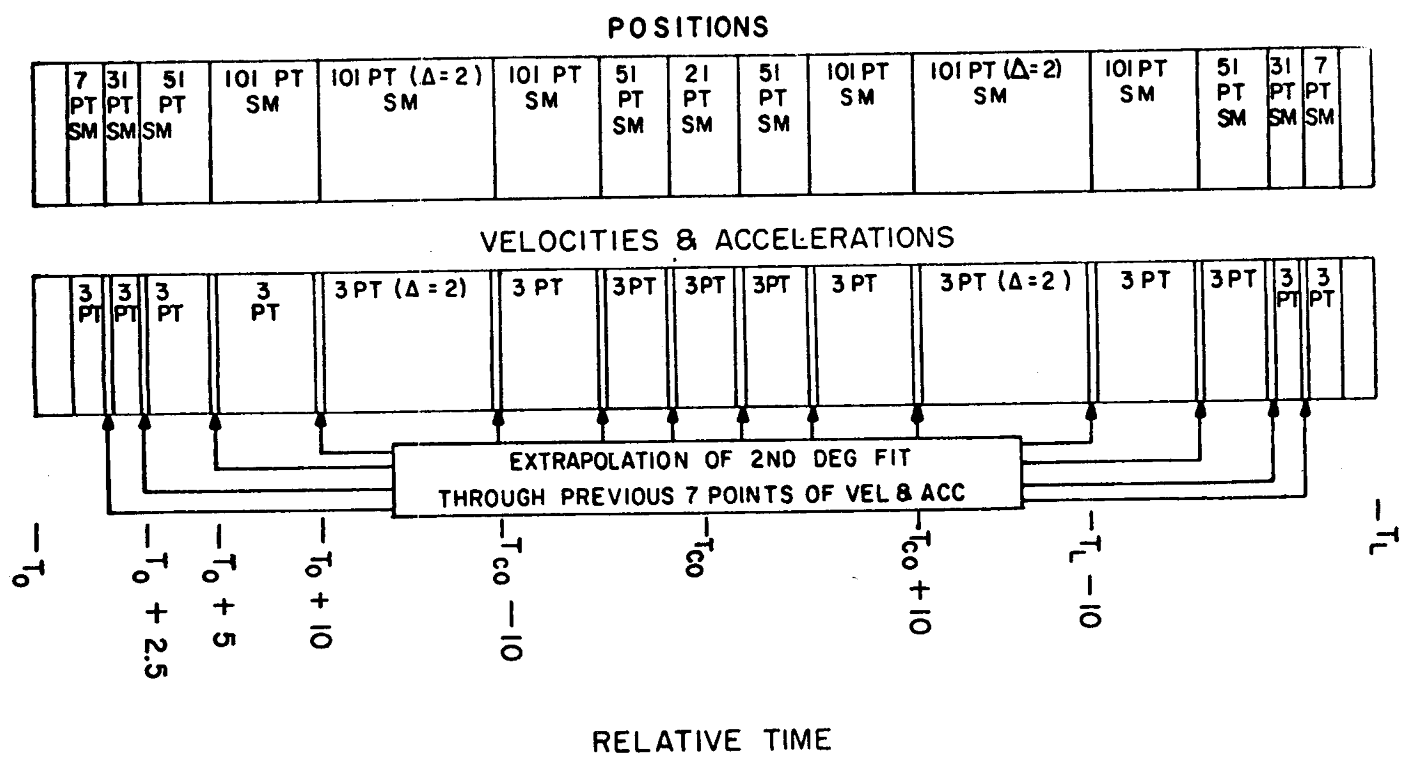

FIG.1 - FIRST PASS OF PROGRAM

[redacted]

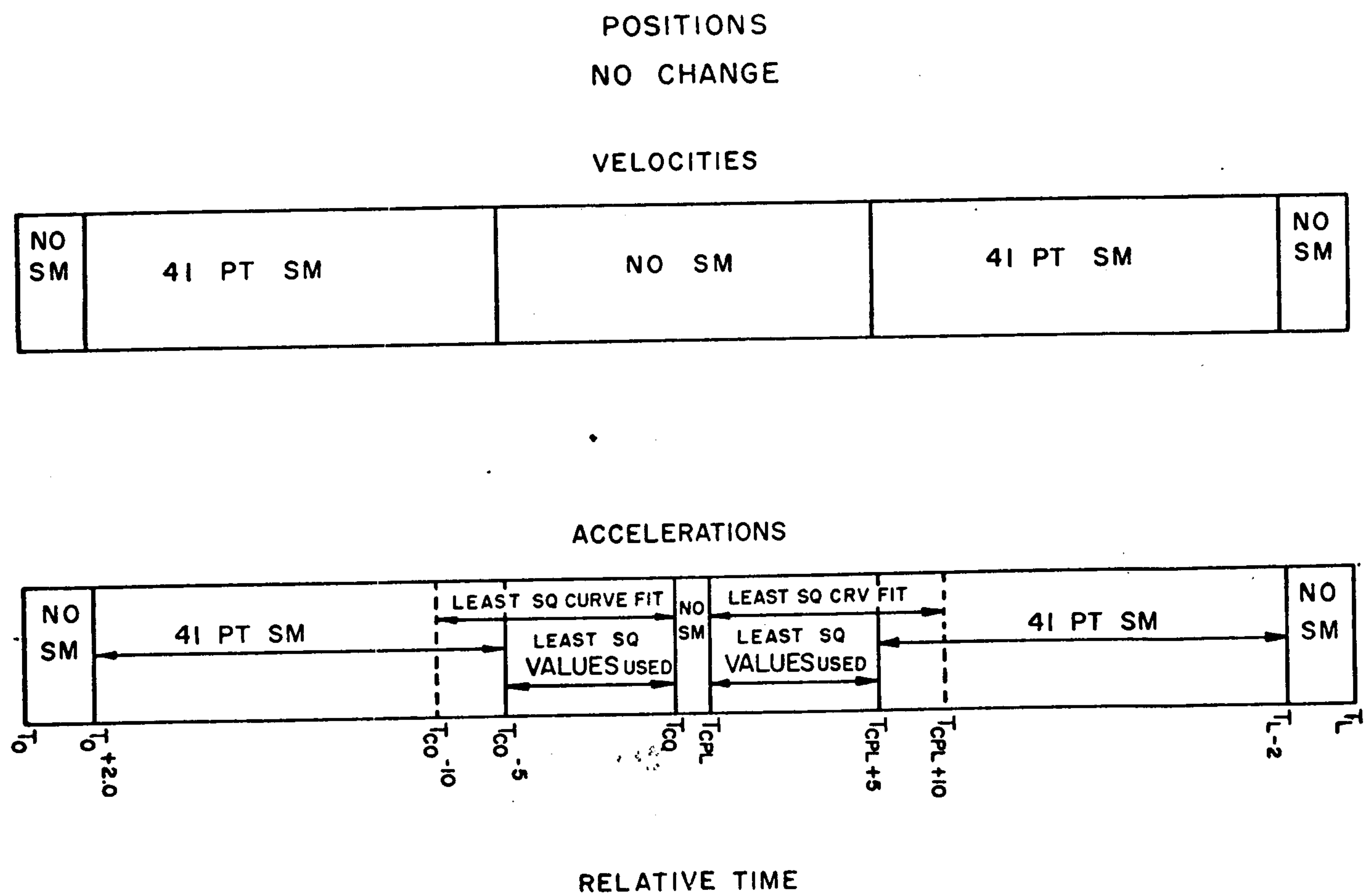

FIG. 2 - SECOND PASS OF PROGRAM

[redacted]

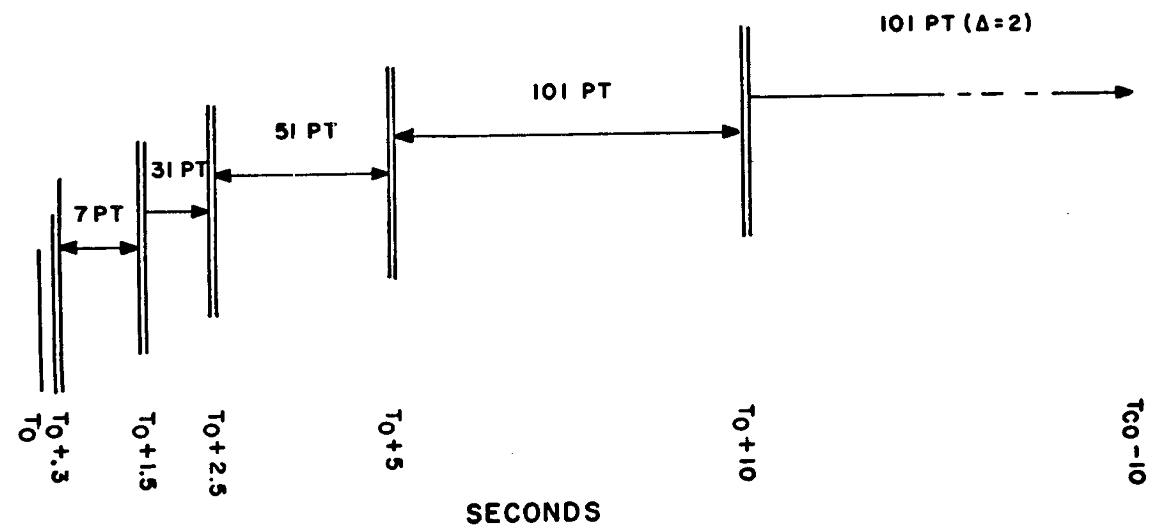

FIG. 3. INITIAL SECTION OF FIRST PASS

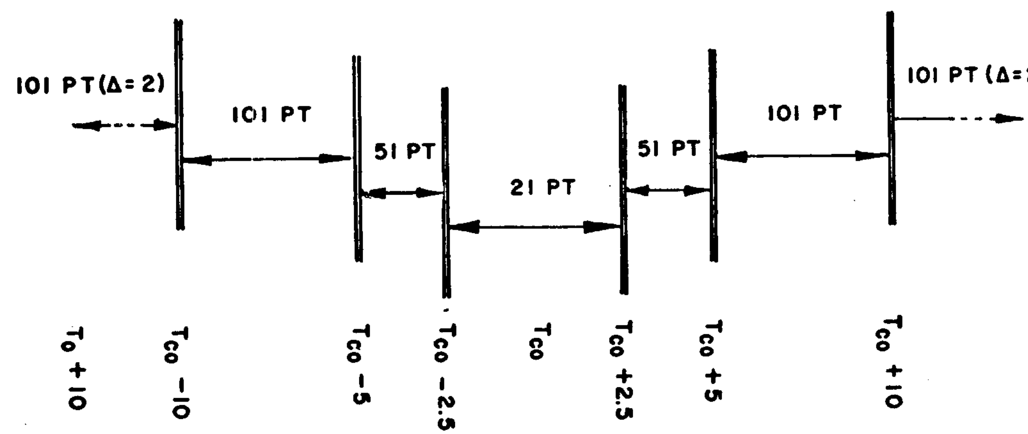

FIG. 4. CUTOFF SECTION OF FIRST PASS

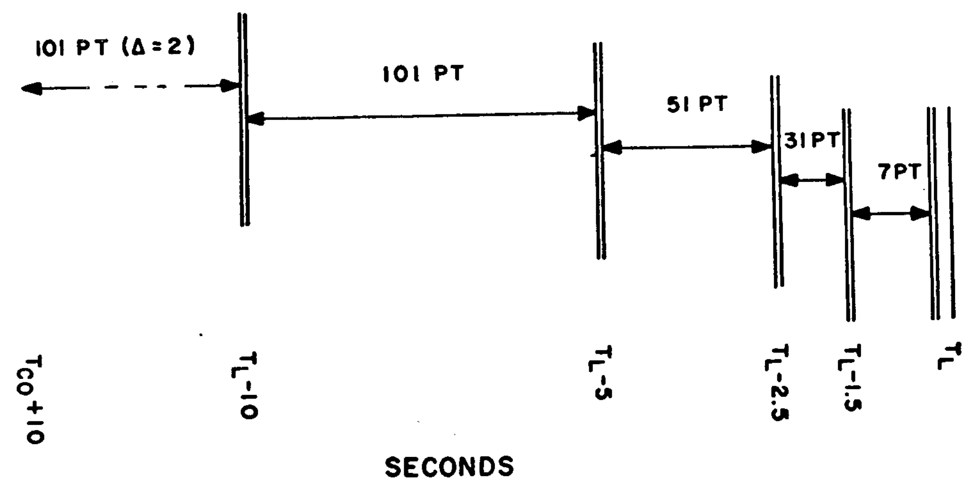

FIG. 5. FINAL SECTION OF FIRST PASS

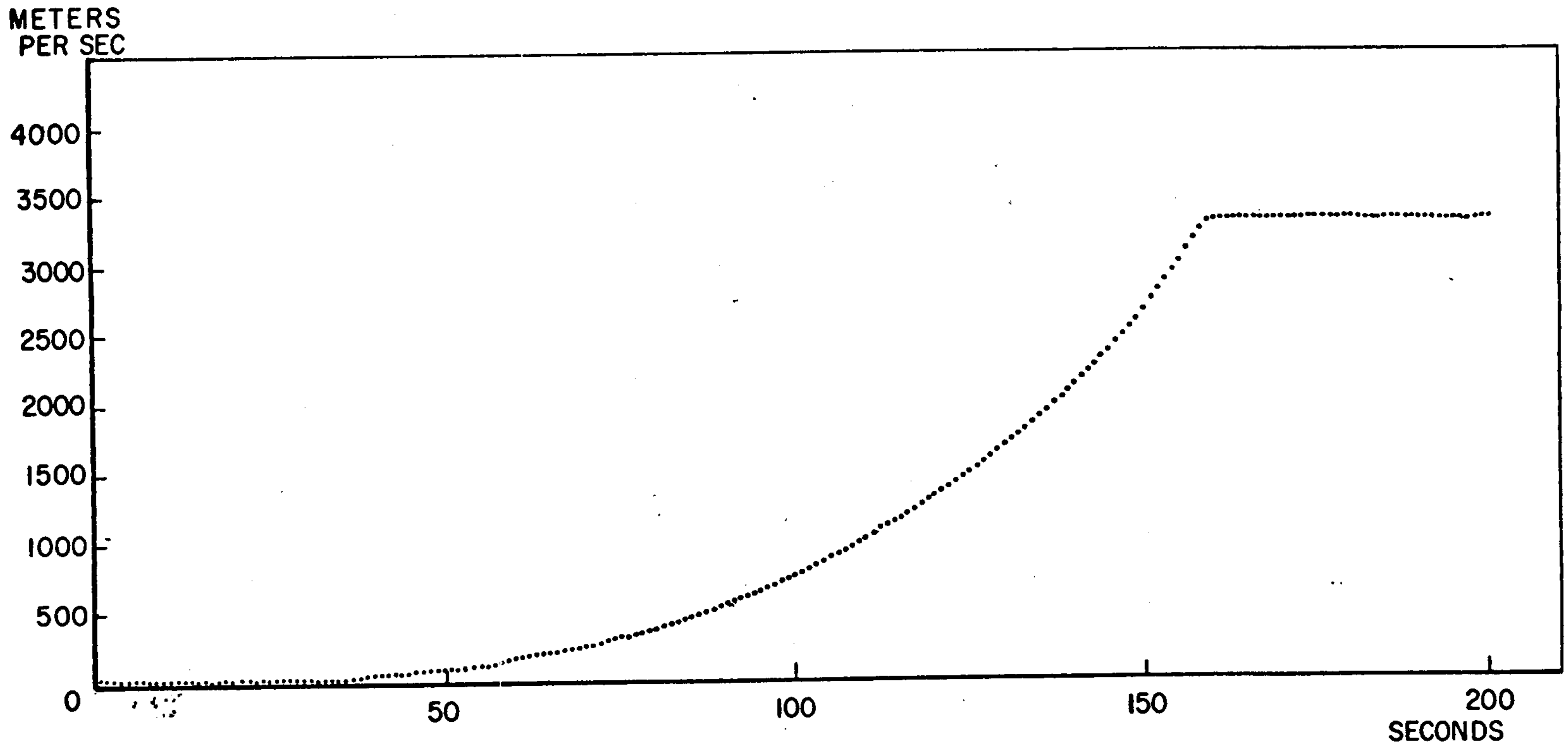

FIG 6. SMOOTH X VELOCITY UDOP (0 to 200 sec)

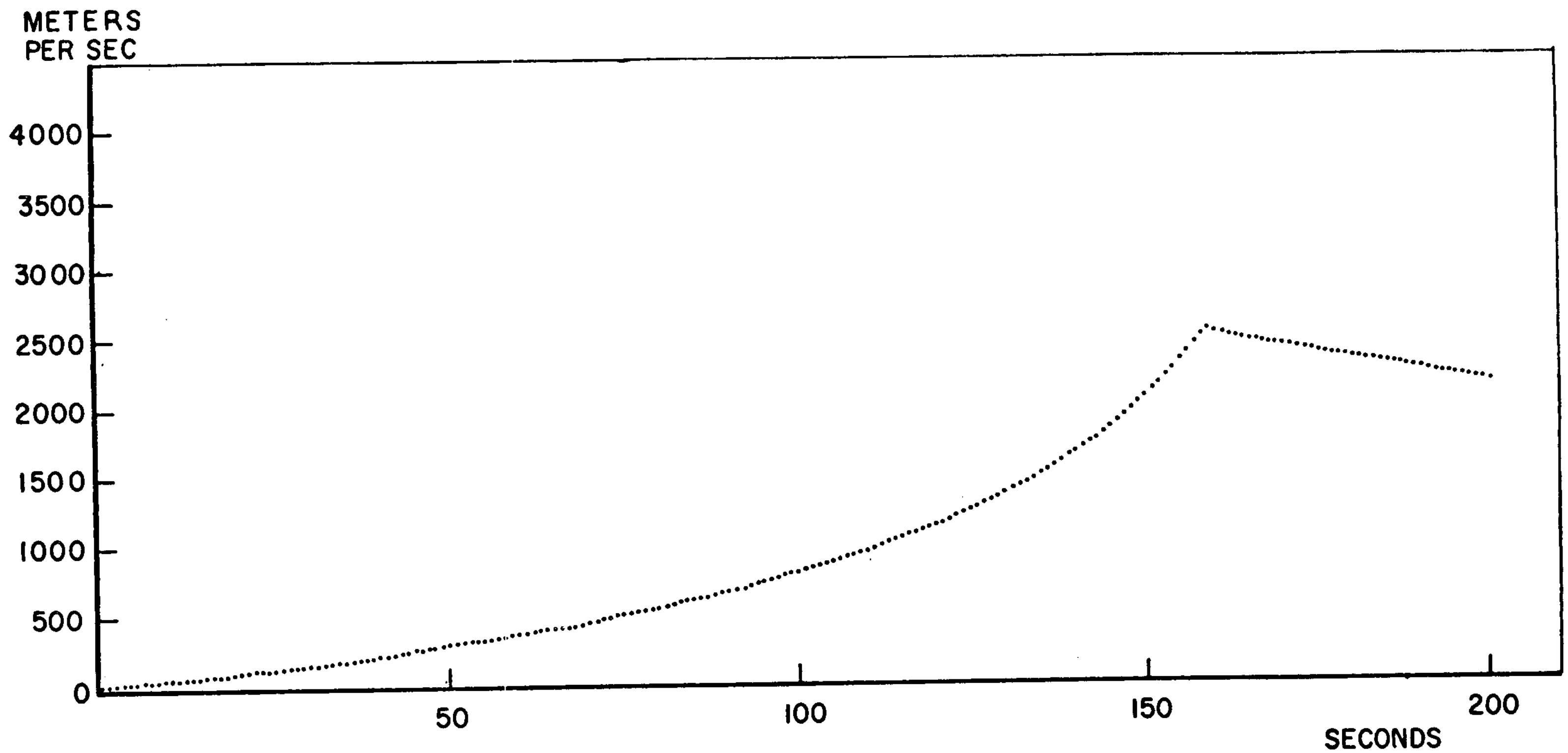

FIG 7. SMOOTH Y VELOCITY UDOP (0 to 200 sec)

[redacted]

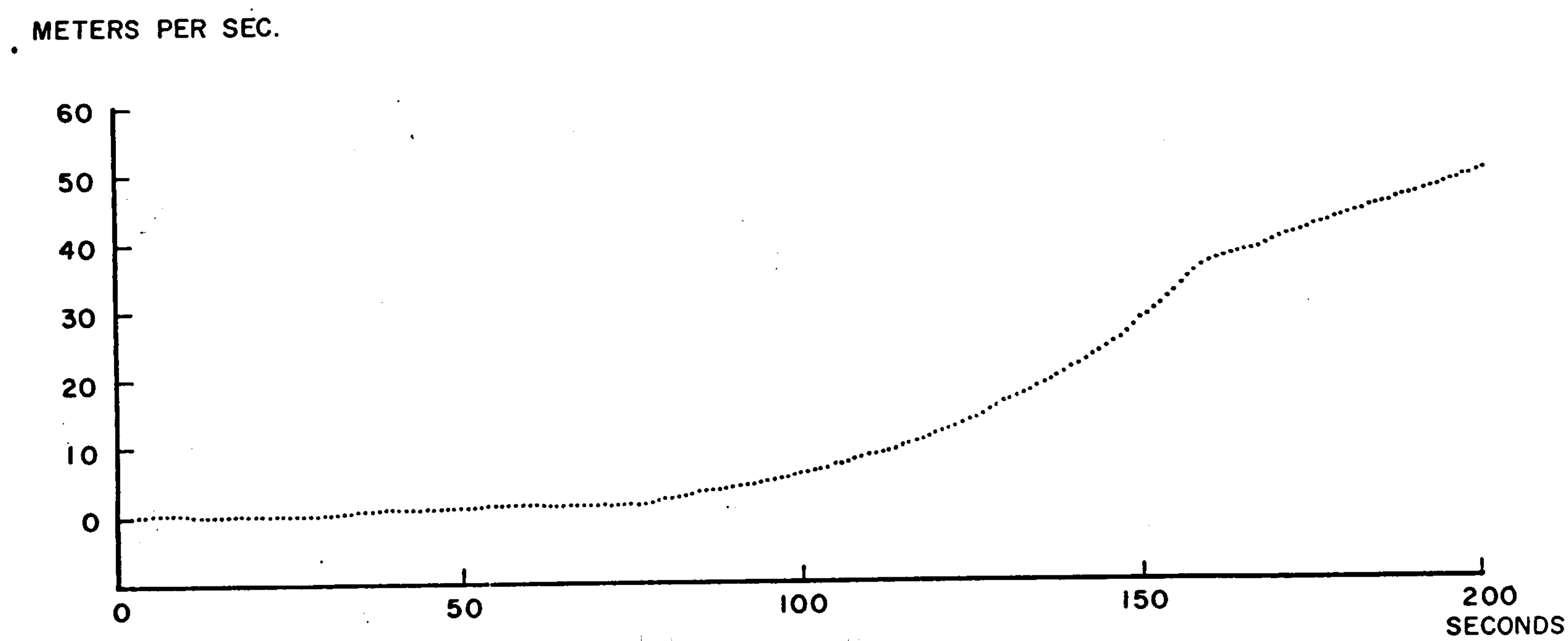

FIG. 8. SMOOTH Z VELOCITY UDOP (0 TO 200 SECONDS)

[redacted]

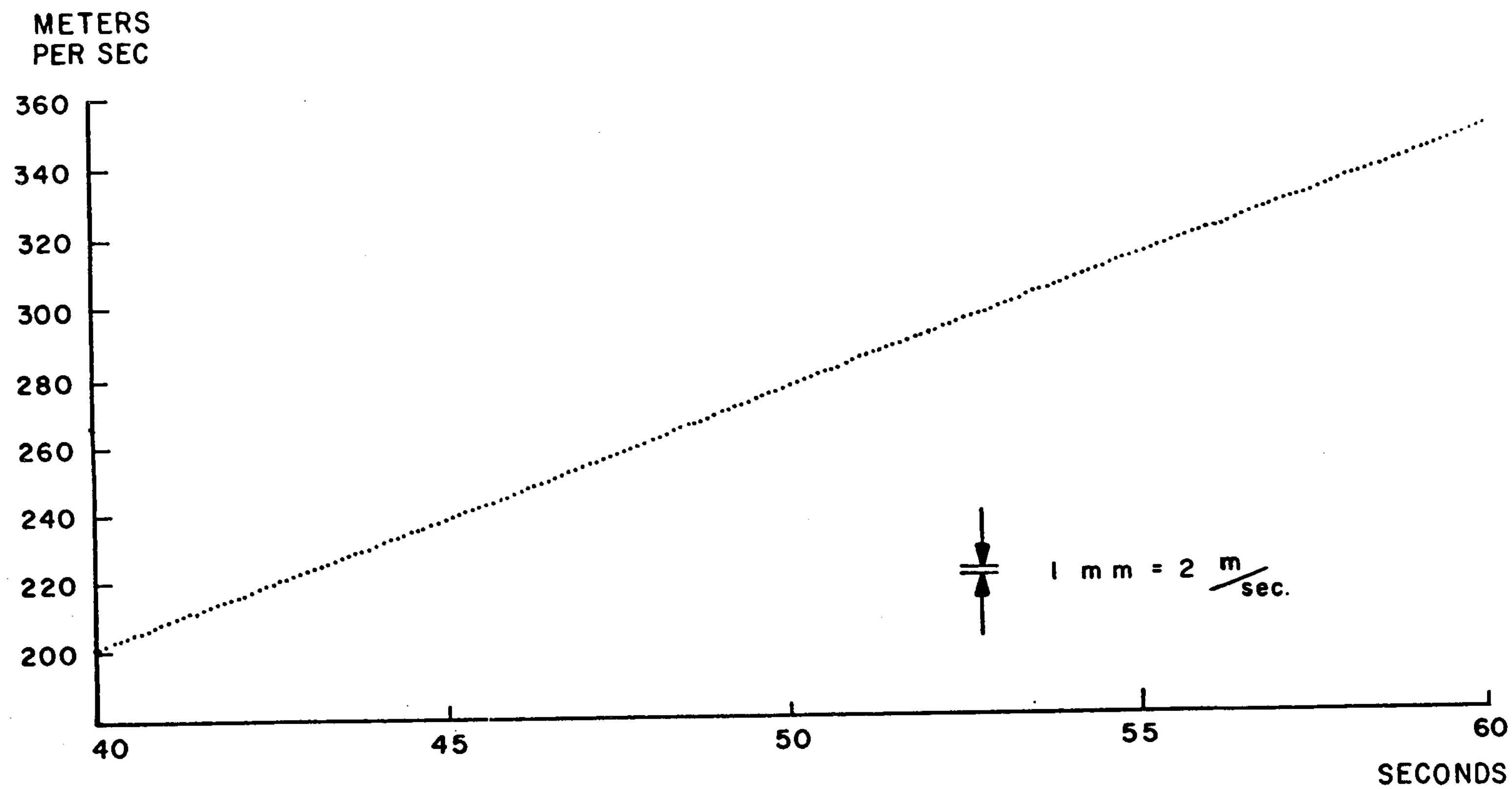

FIG. 9. SMOOTH Y VELOCITY UDOP (40 TO 60 SECONDS)

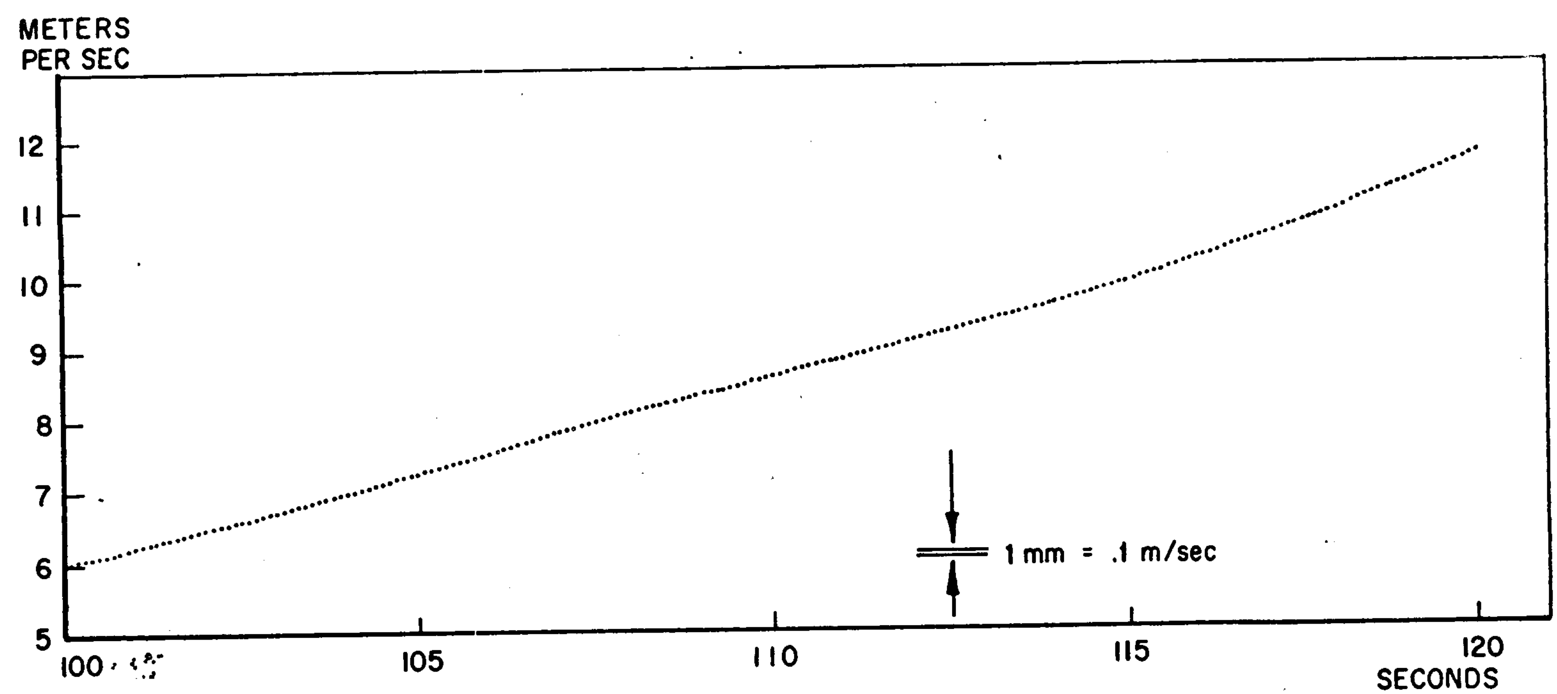

FIG 10. SMOOTH Z VELOCITY UDOP (100 to 120 sec)

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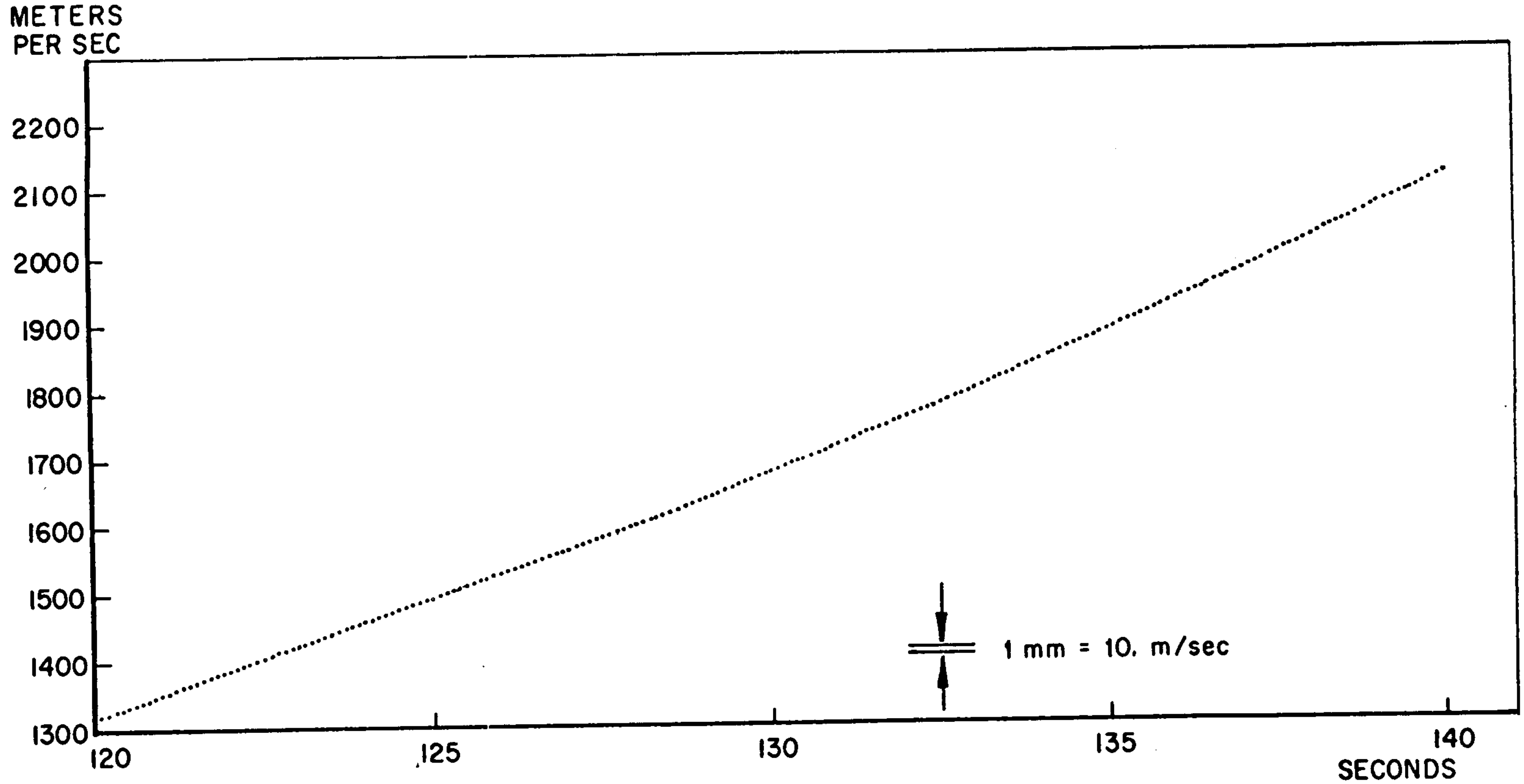

FIG 11. SMOOTH X VELOCITY UDOP (120 to 140 sec)

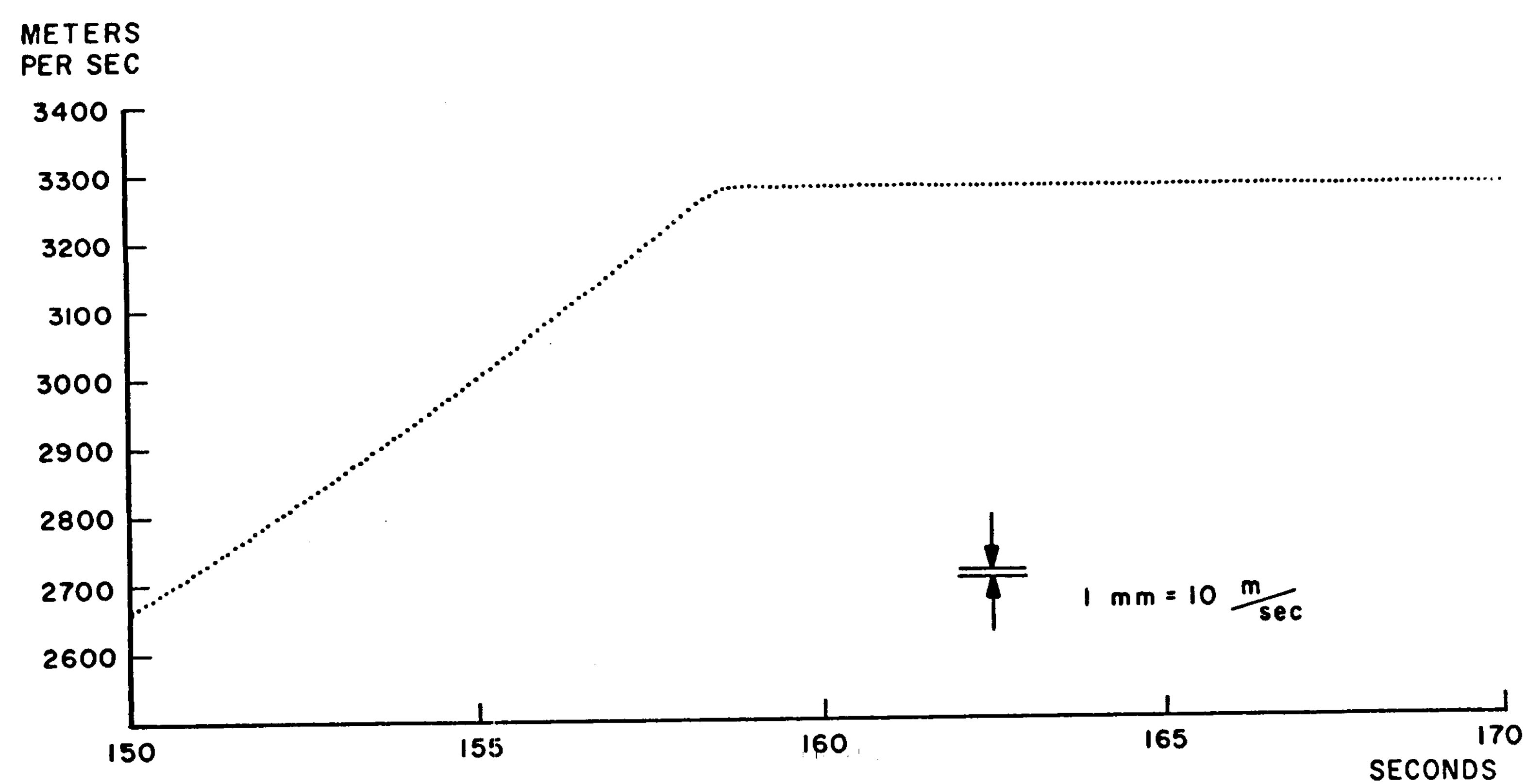

FIG. 12. SMOOTH X VELOCITY UDOP (150 TO 170 SECONDS)

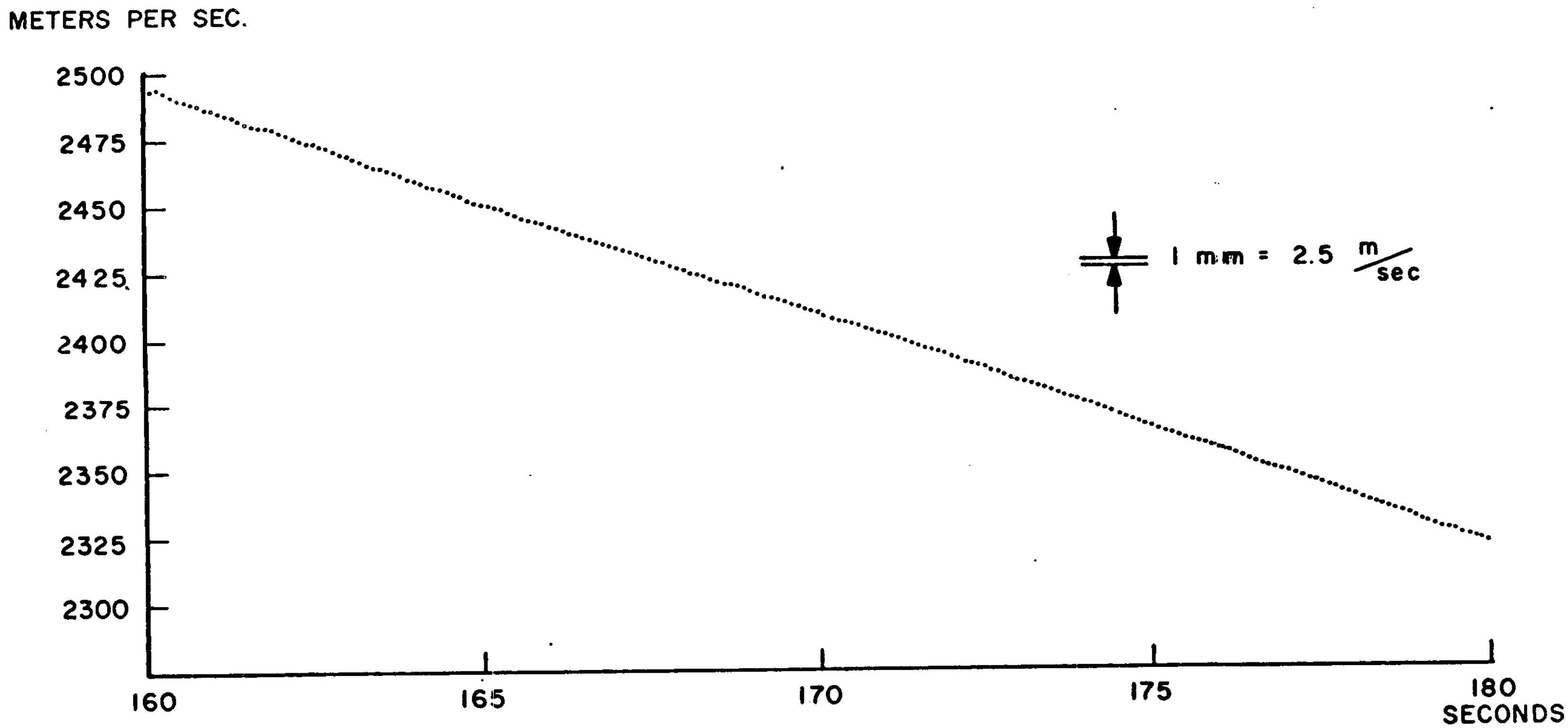

FIG. 13. SMOOTH Y VELOCITY UDOP (160 TO 180 SECONDS)

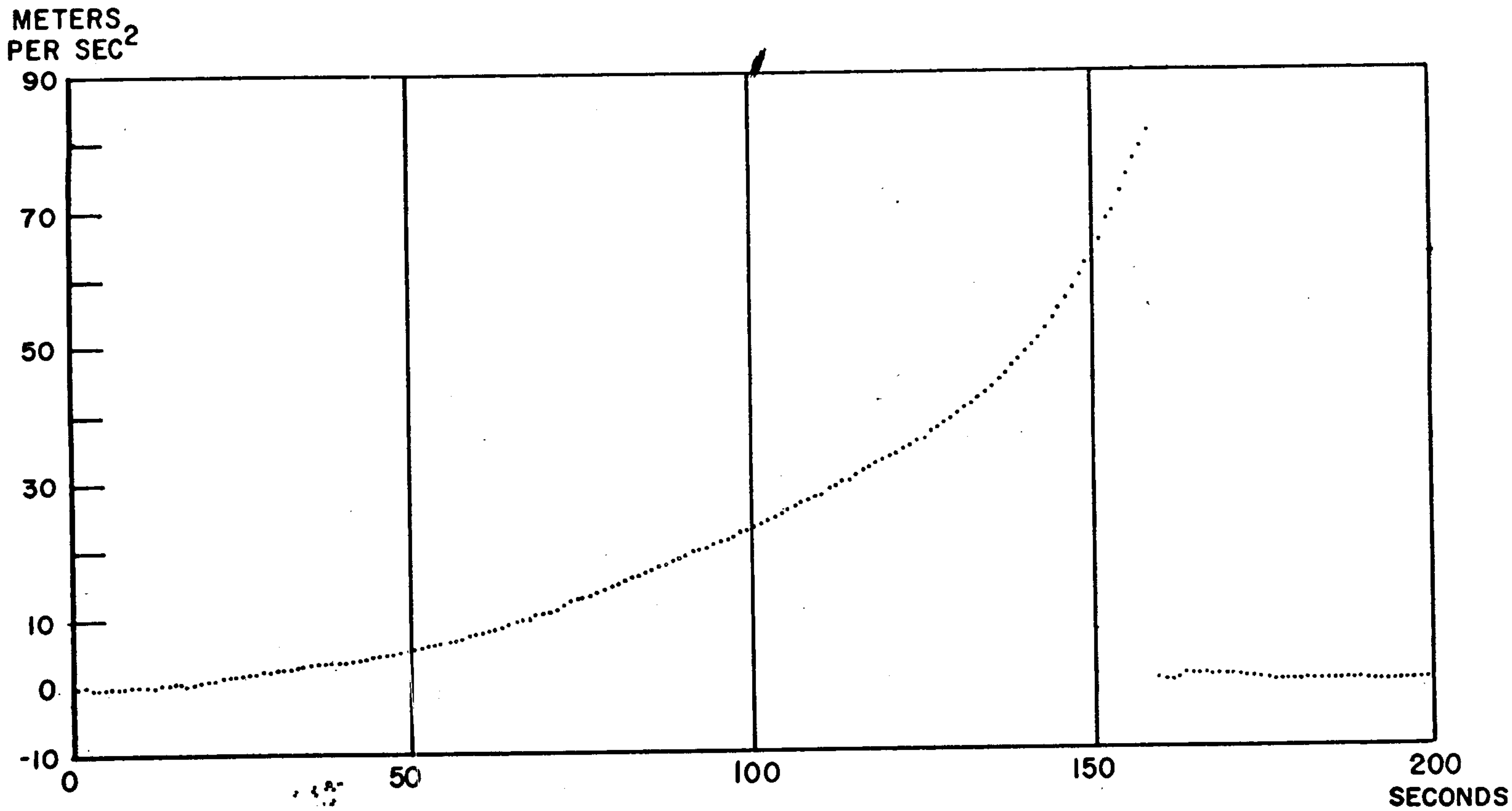

FIG 14 SMOOTH X ACCELERATION UDOP (0 TO 200 SECONDS)

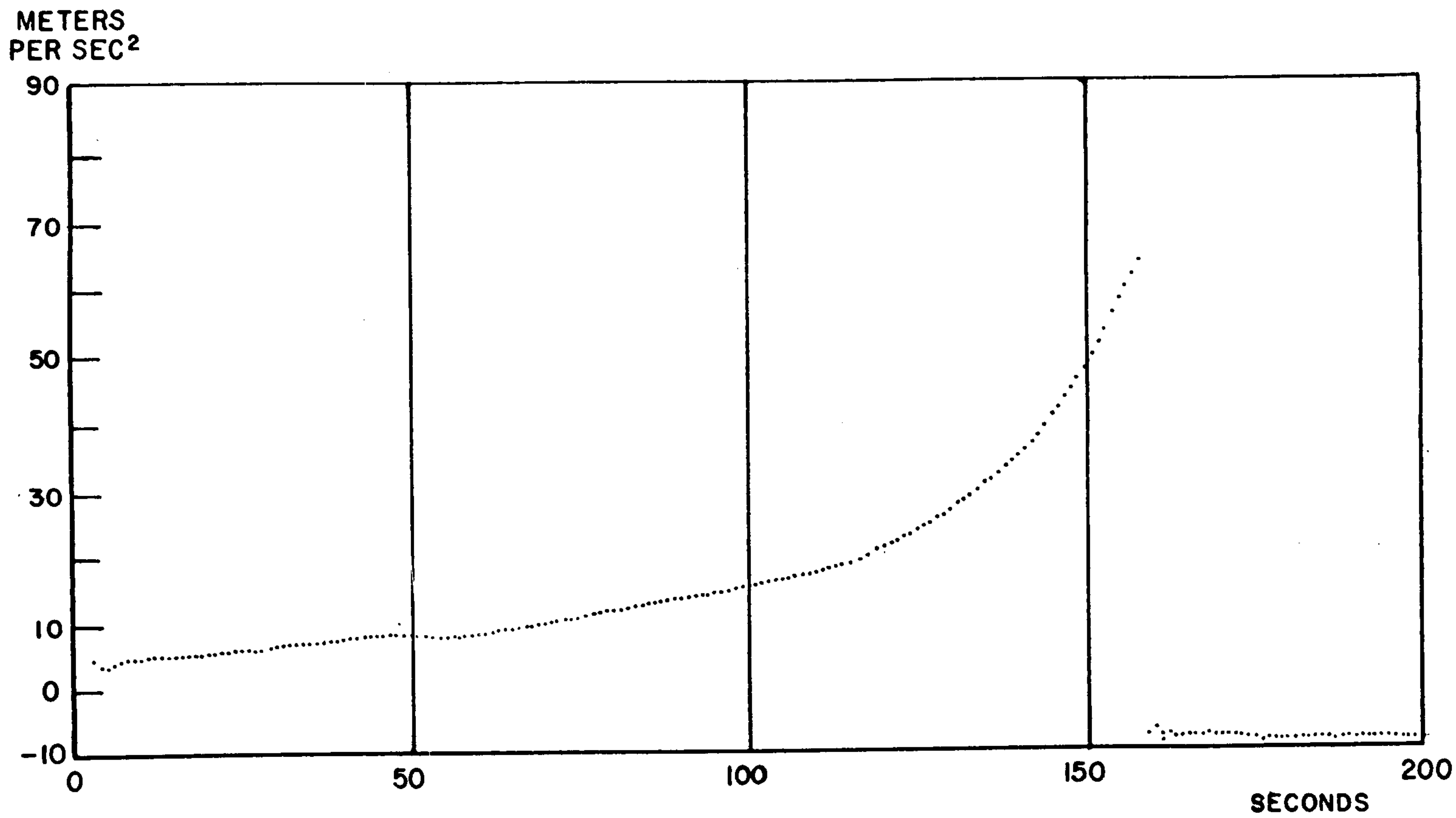

FIG 15 SMOOTH Y ACCELERATION UDOP (0 TO 200 SECONDS)

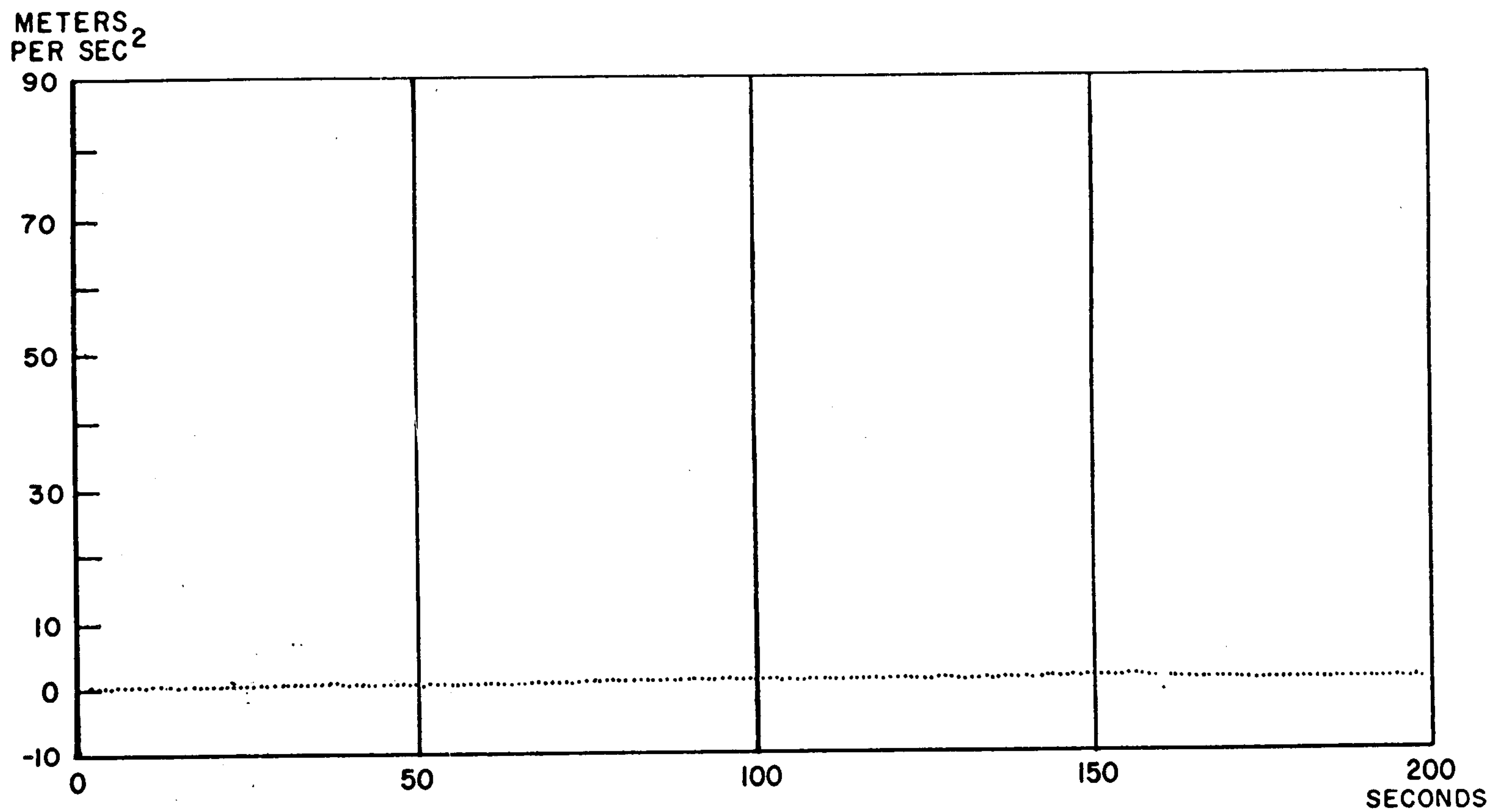

FIG 16 SMOOTH Z ACCELERATION UDOP (0 TO 200 SECONDS)

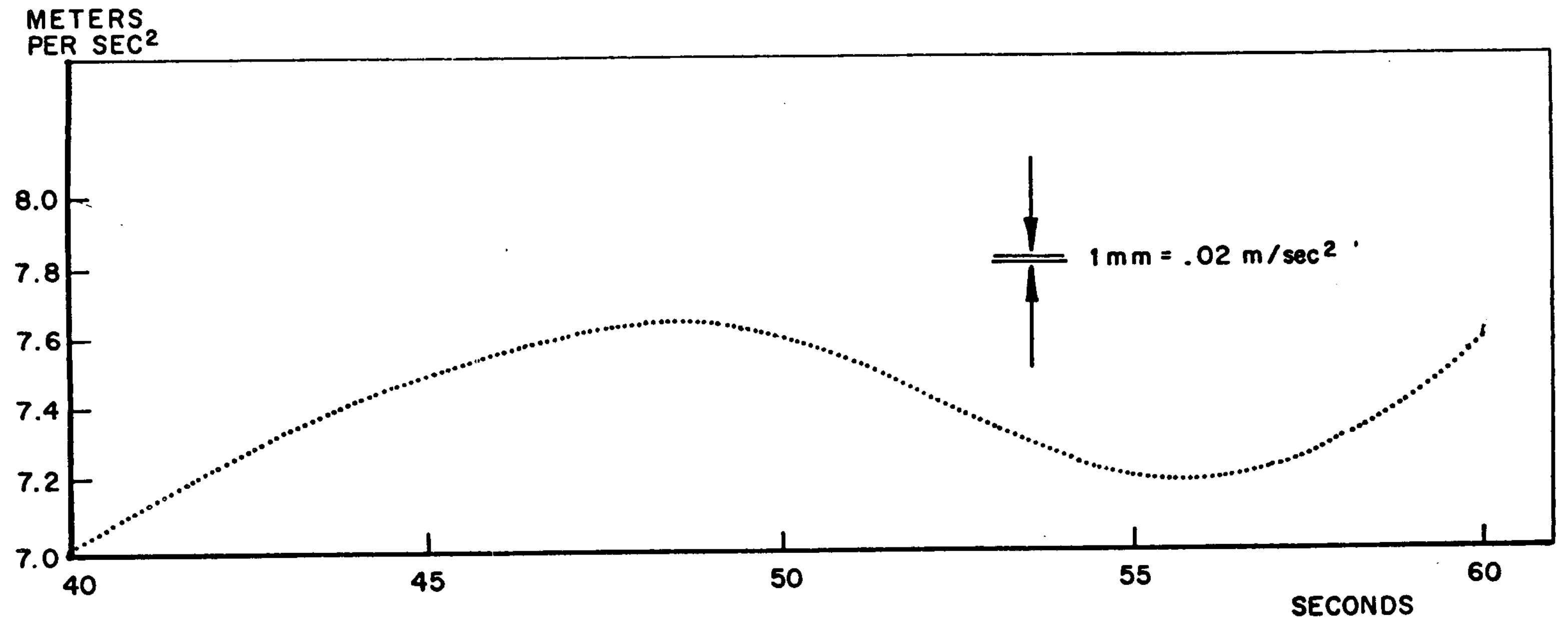

FIG 17. SMOOTH Y ACCELERATION UDOP (40 to 60 sec)

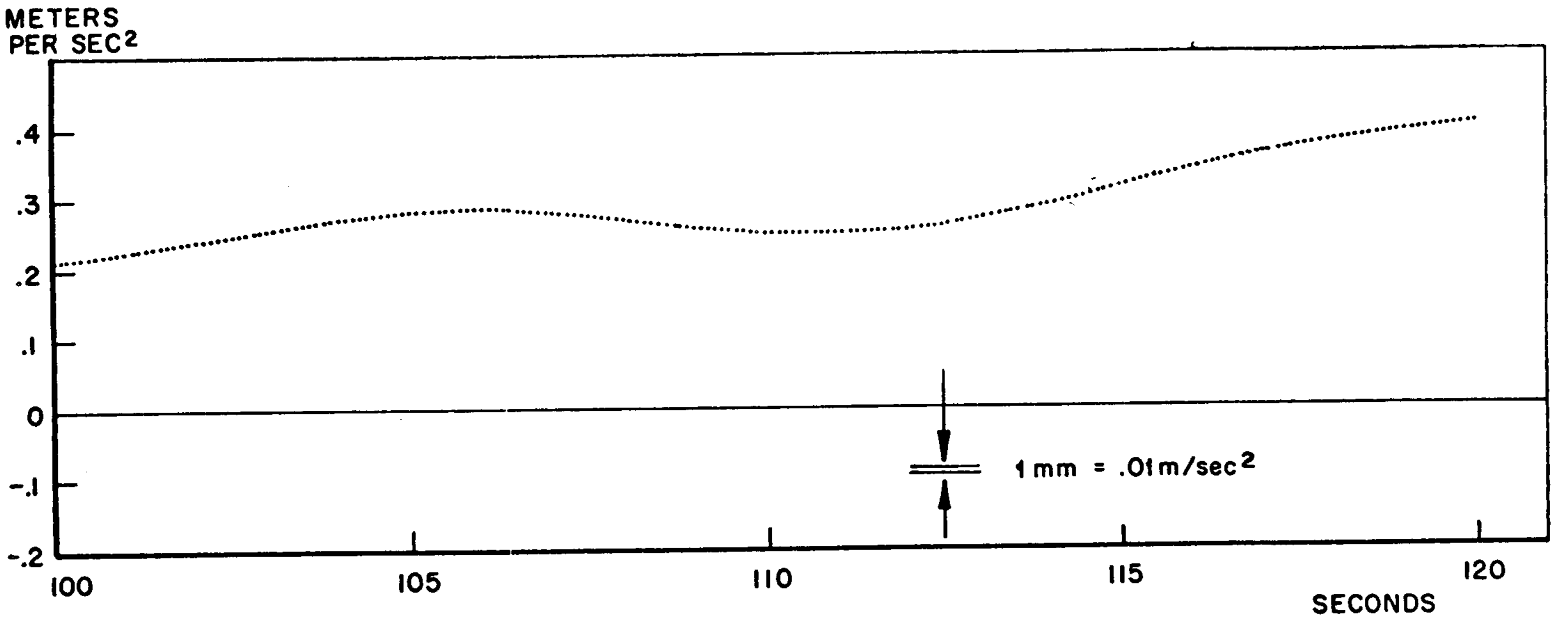

FIG 18. SMOOTH Z ACCELERATION UDOP (100 to 120 sec)

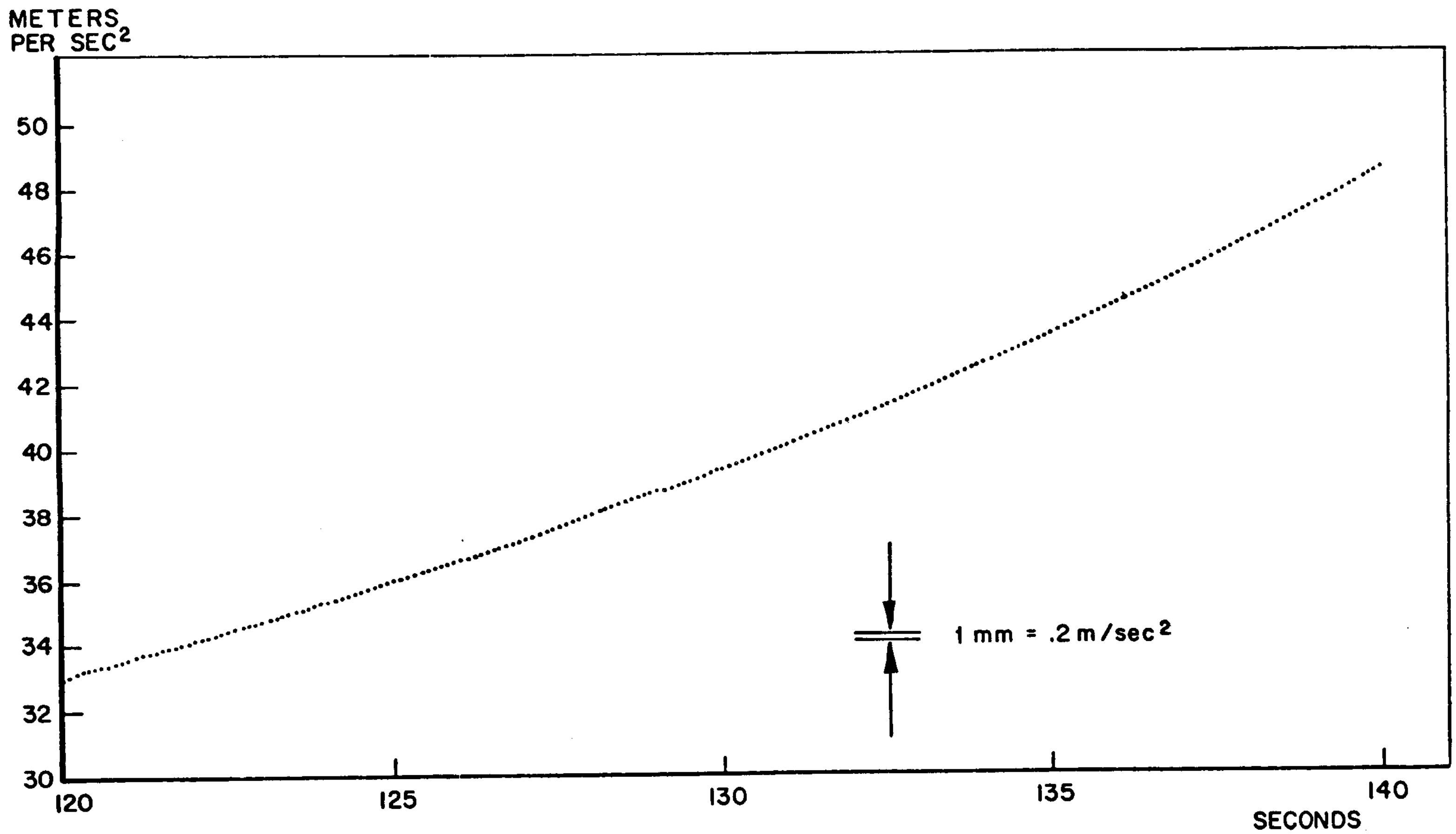

FIG 19. SMOOTH X ACCELERATION UDOP (120 to 140 sec)

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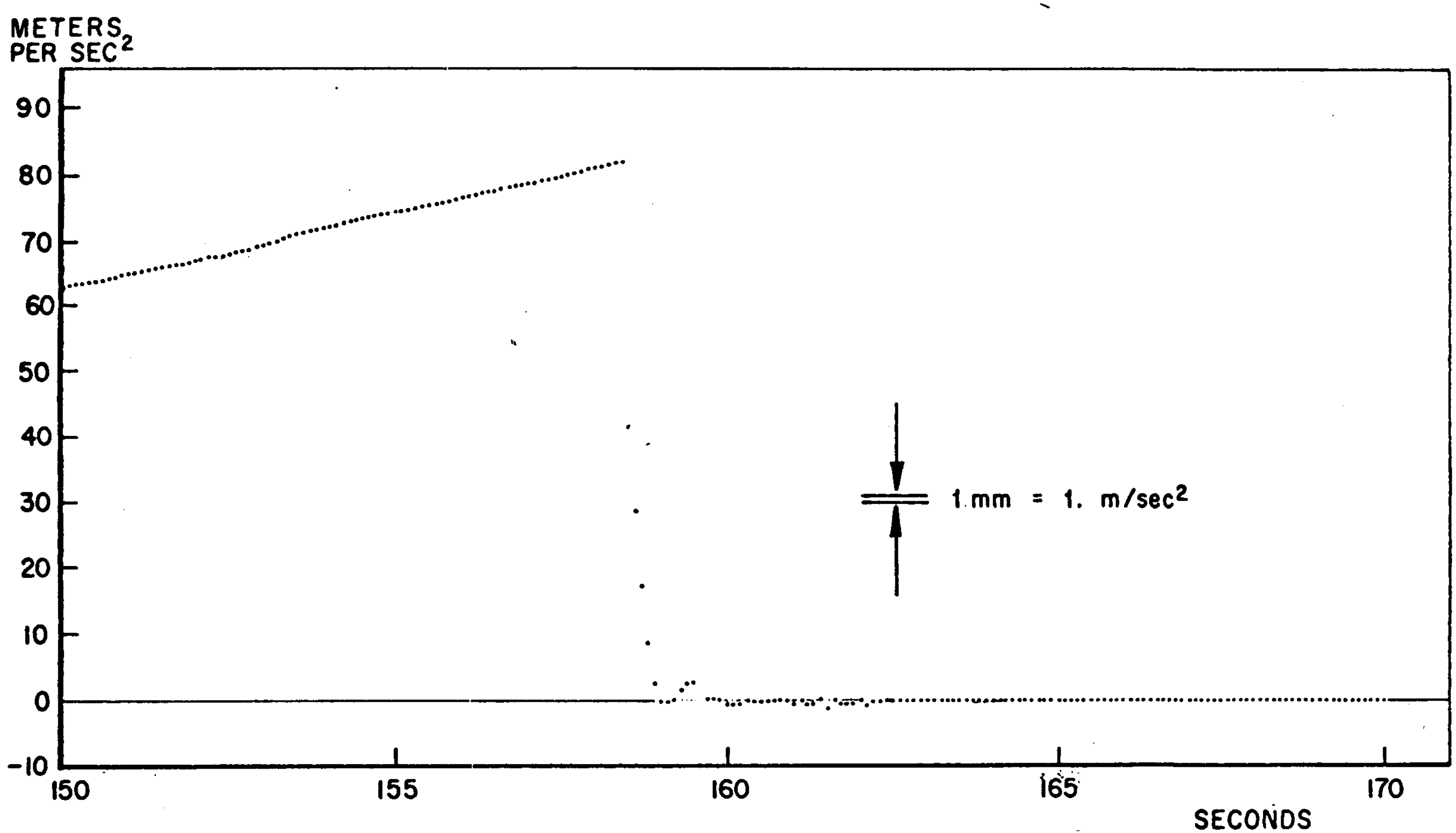

FIG 20. SMOOTH X ACCELERATION UDOP (150 to 170 sec)

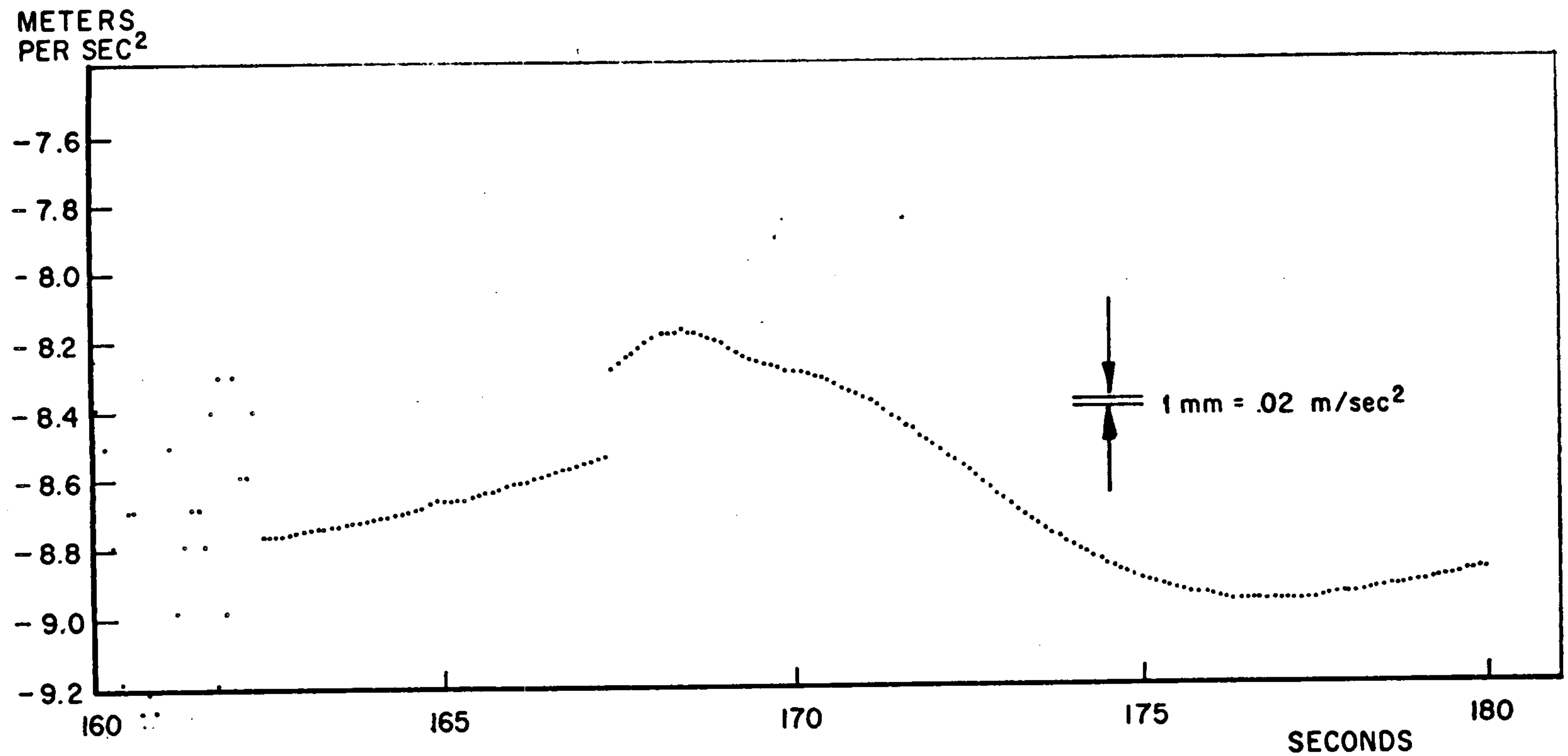

FIG 21. SMOOTH Y ACCELERATION UDOP (160 to 180 sec)

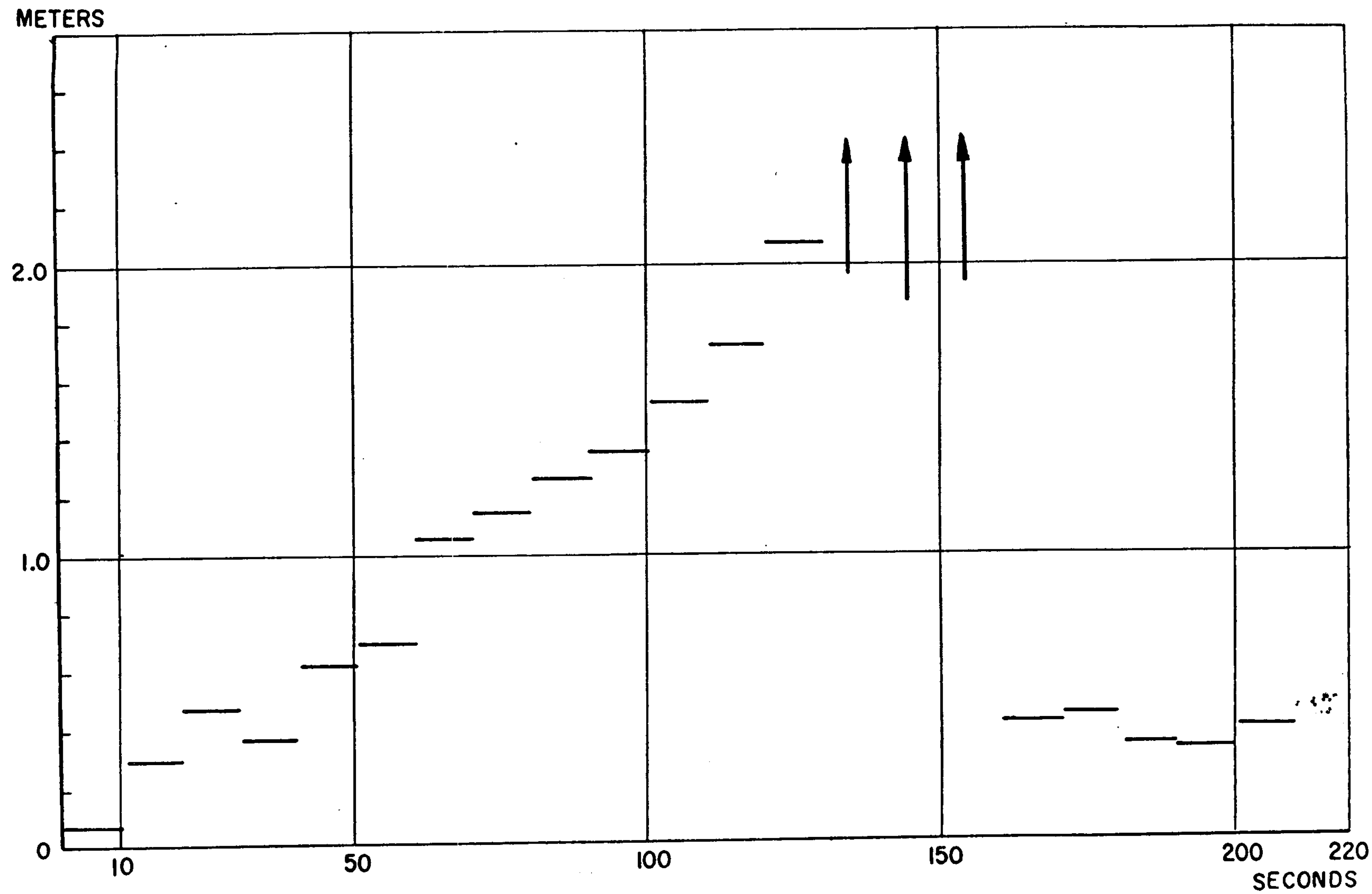

FIG. 22 STANDARD DEVIATION OF SMOOTH X POSITIONS UDOP

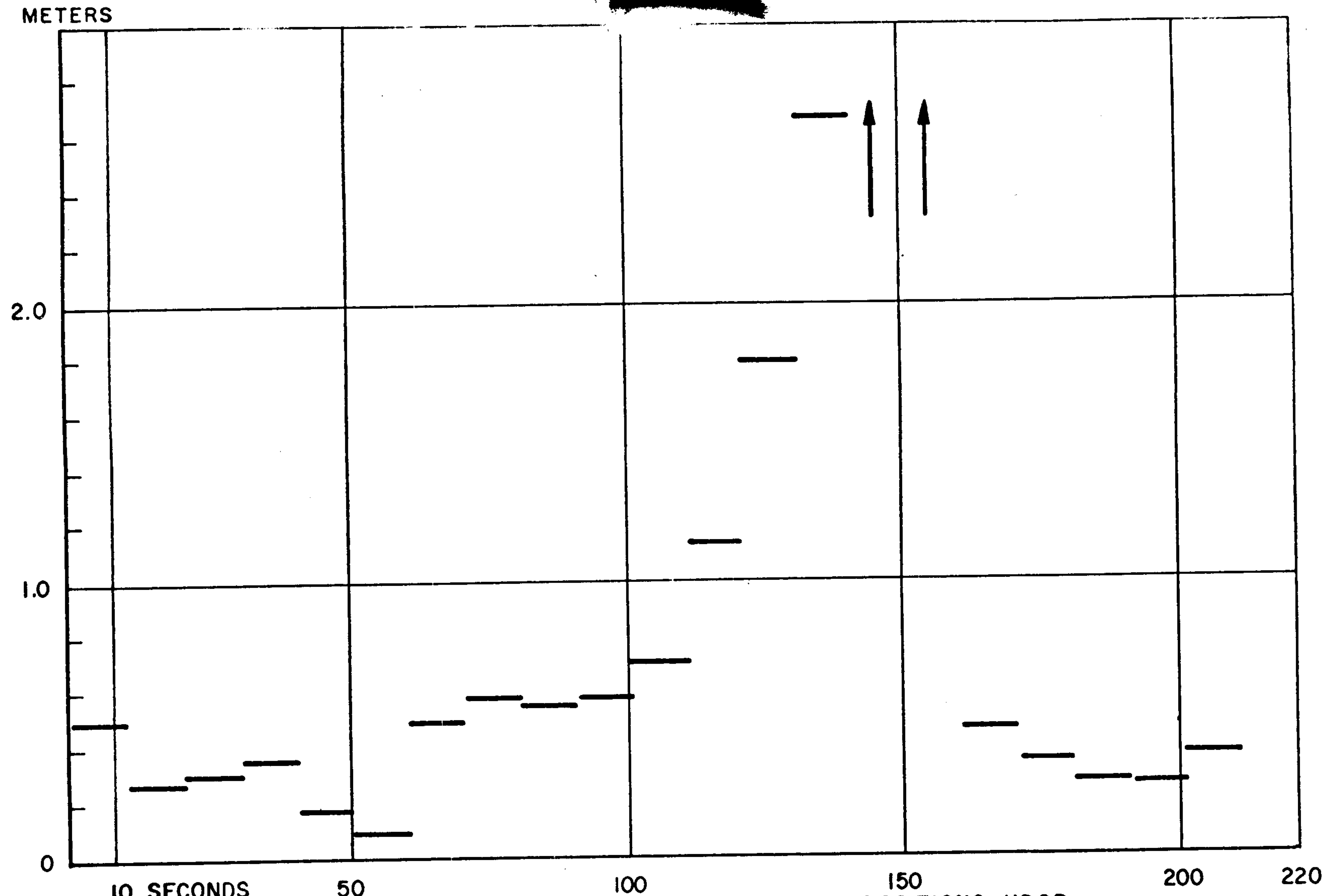

FIG 23 STANDARD DEVIATION OF SMOOTH Y POSITIONS UDOP

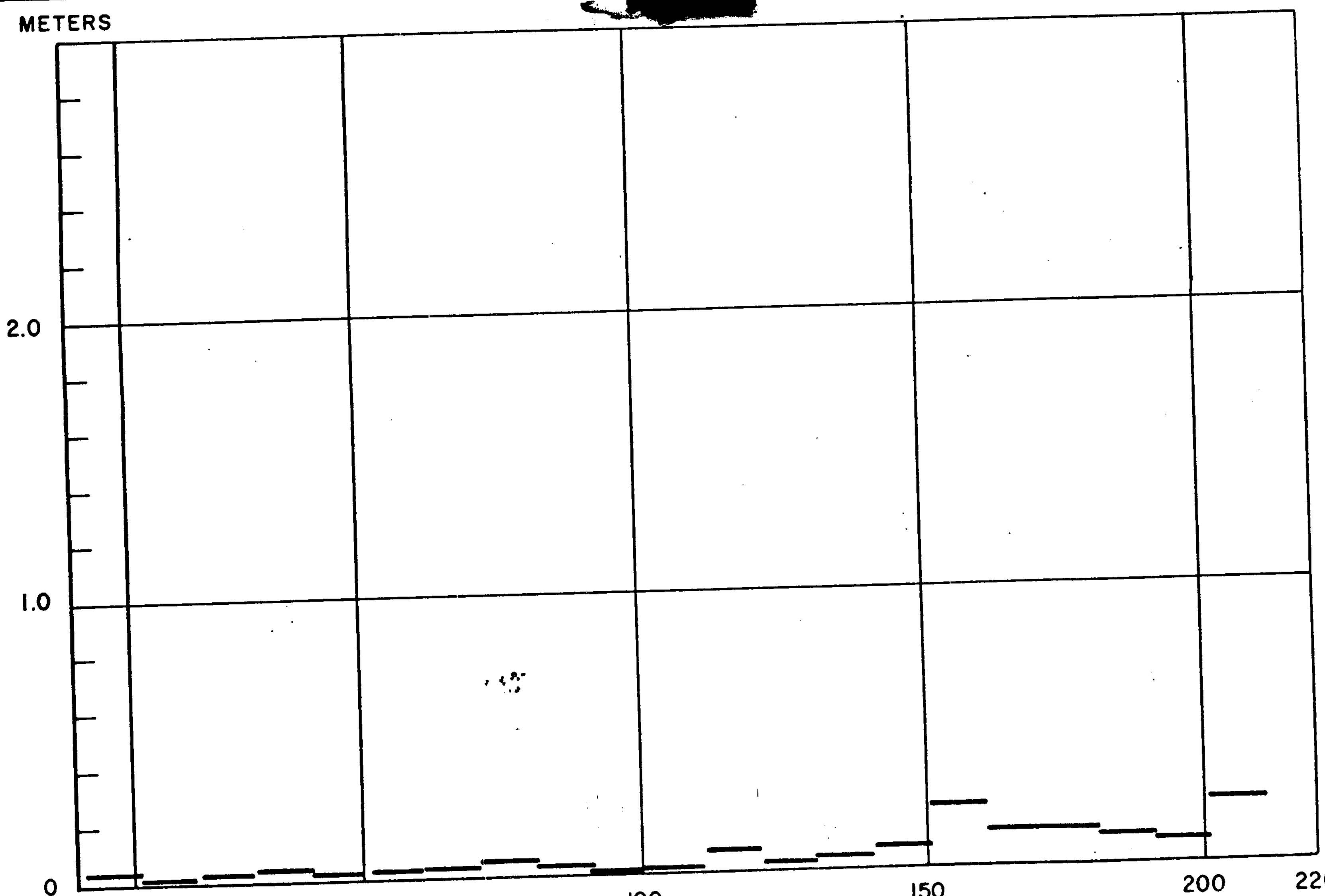

FIG 24 STANDARD DEVIATION OF SMOOTH Z POSITIONS UDOP

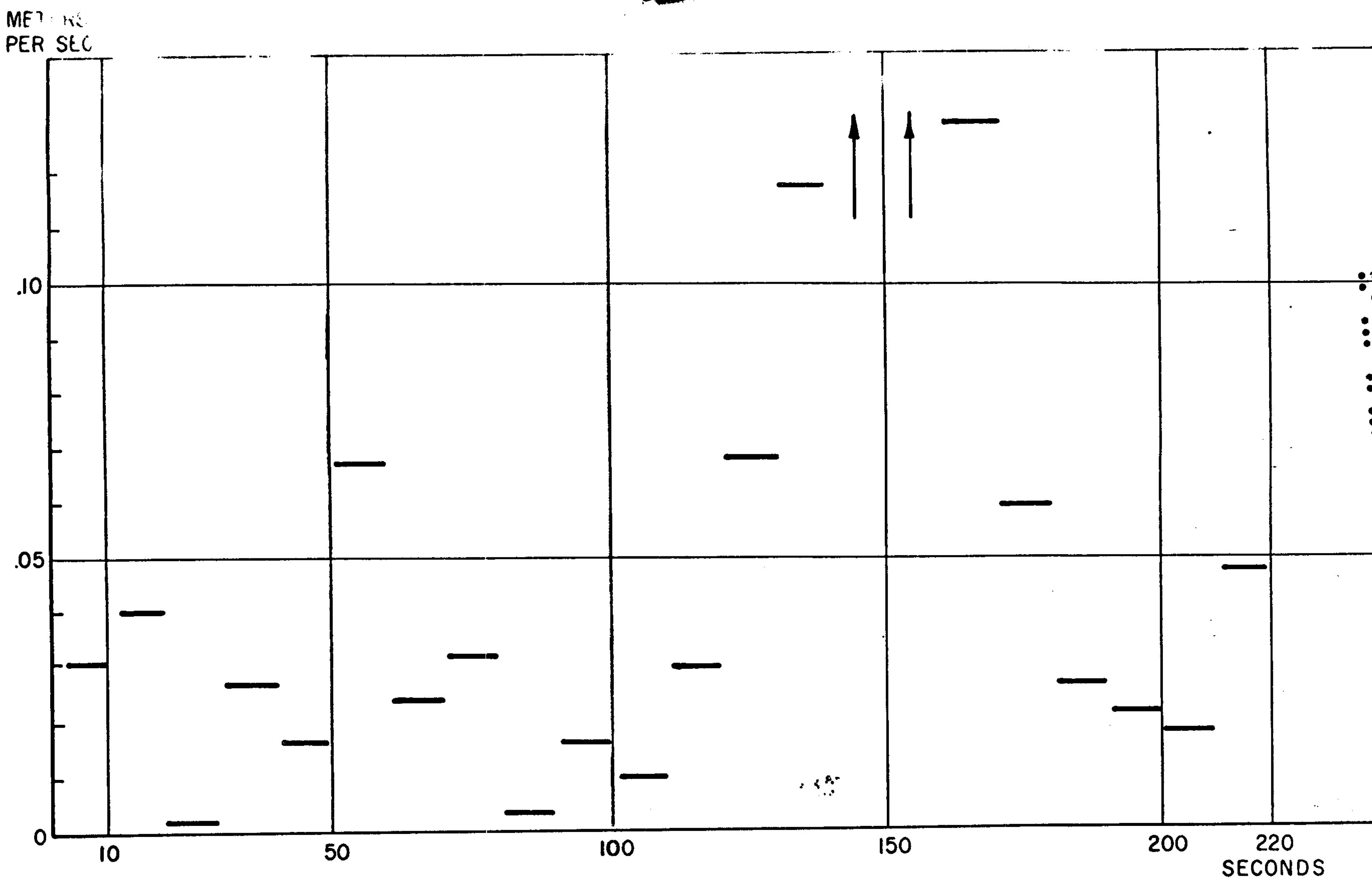

FIG 25 STANDARD DEVIATION OF SMOOTH X VELOCITIES UDOP

[redacted]

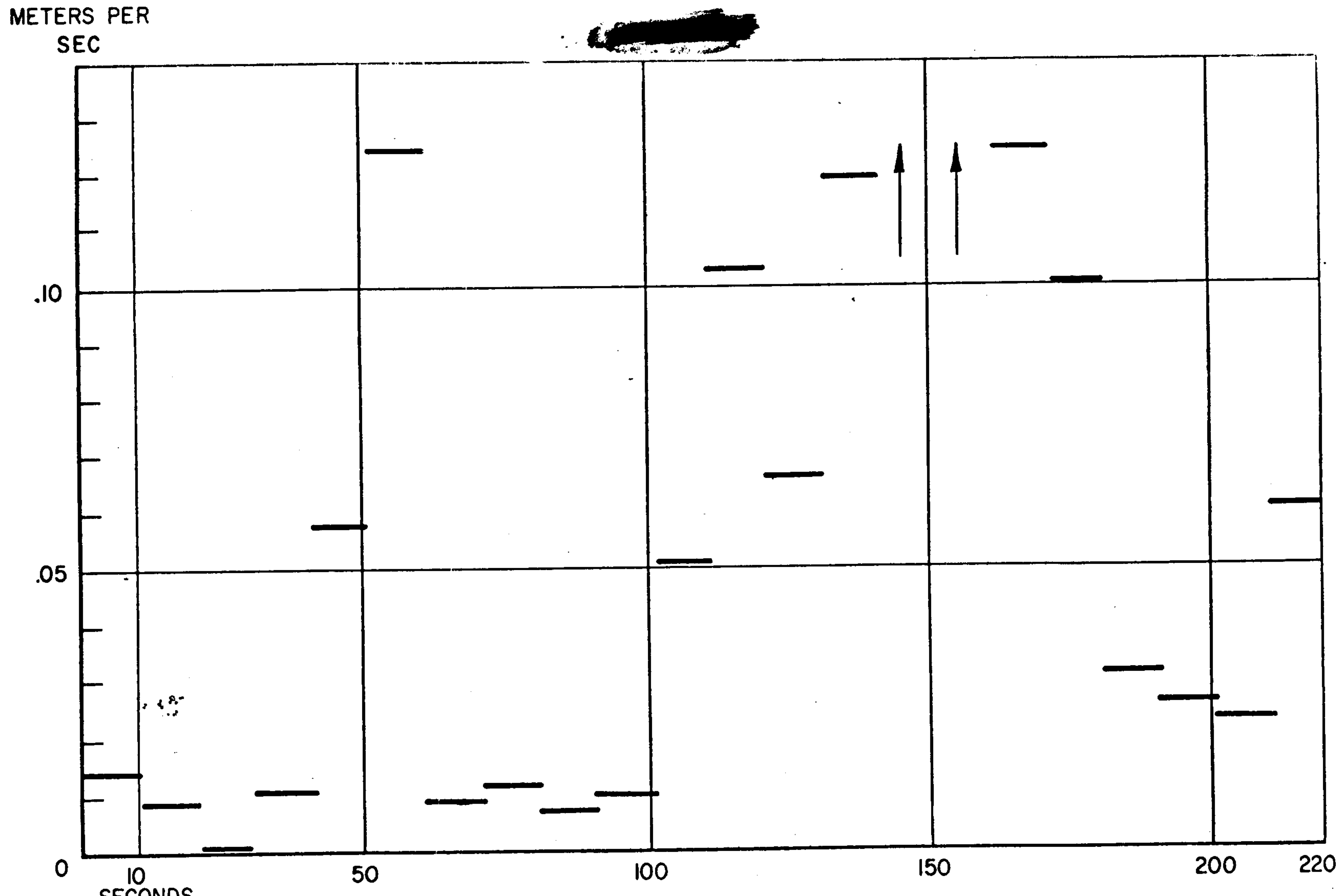

FIG 26 STANDARD DEVIATION OF SMOOTH Y VELOCITIES UDOP

[redacted]

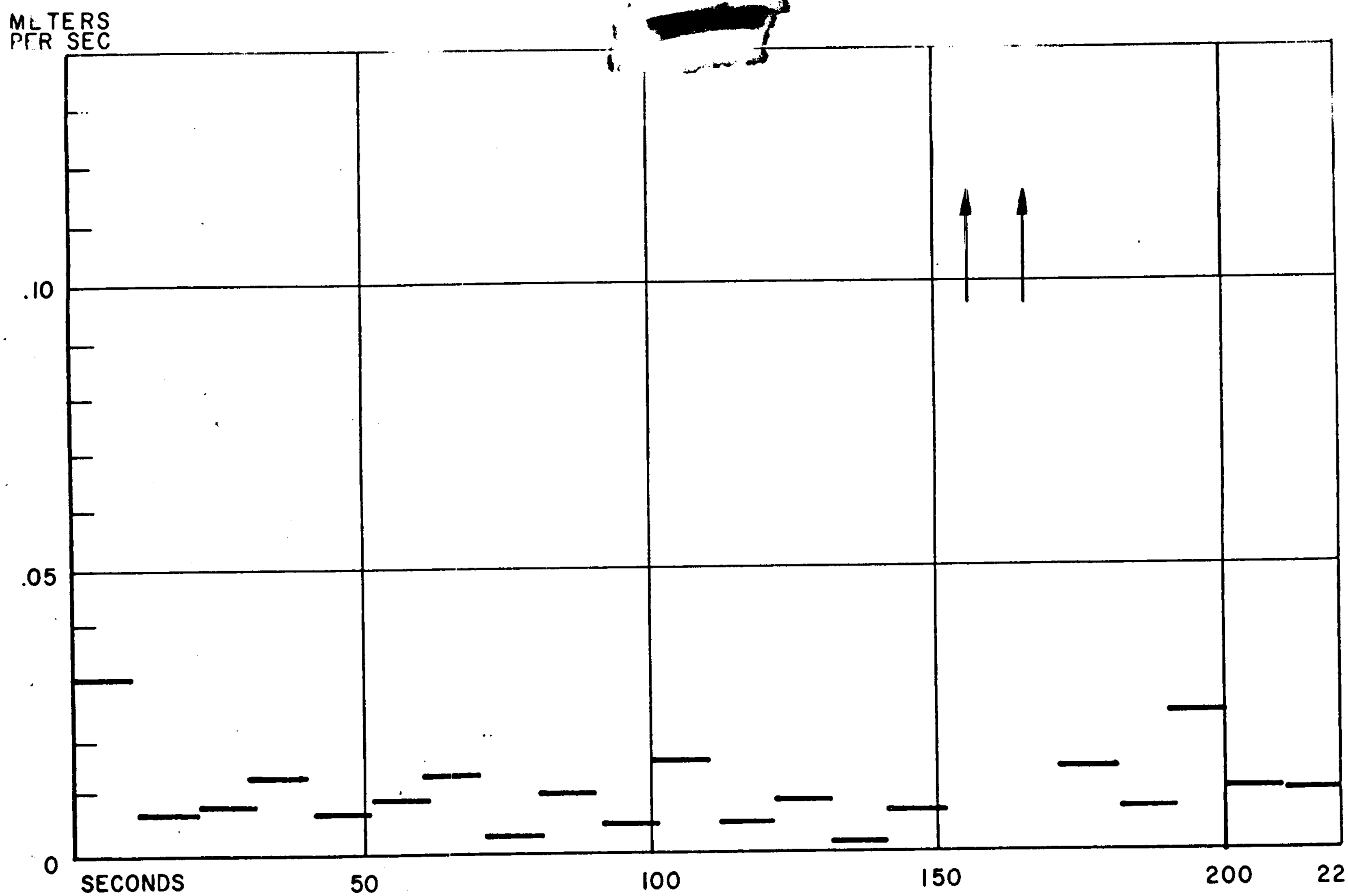

FIG 27 STANDARD DEVIATION OF SMOOTH Z VELOCITIES UDOP

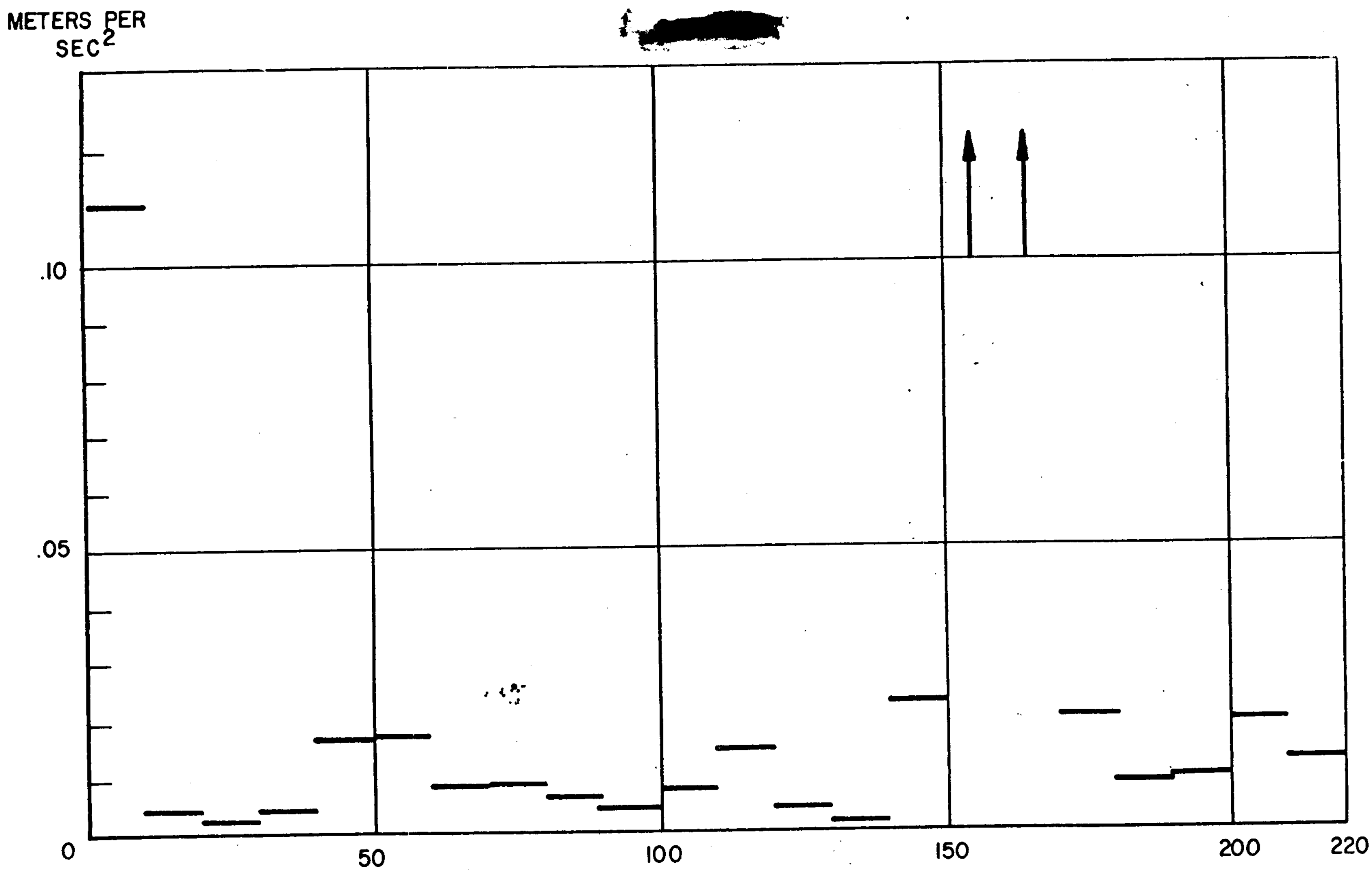

FIG 28 STANDARD DEVIATION OF SMOOTH X ACCELERATIONS UDOP

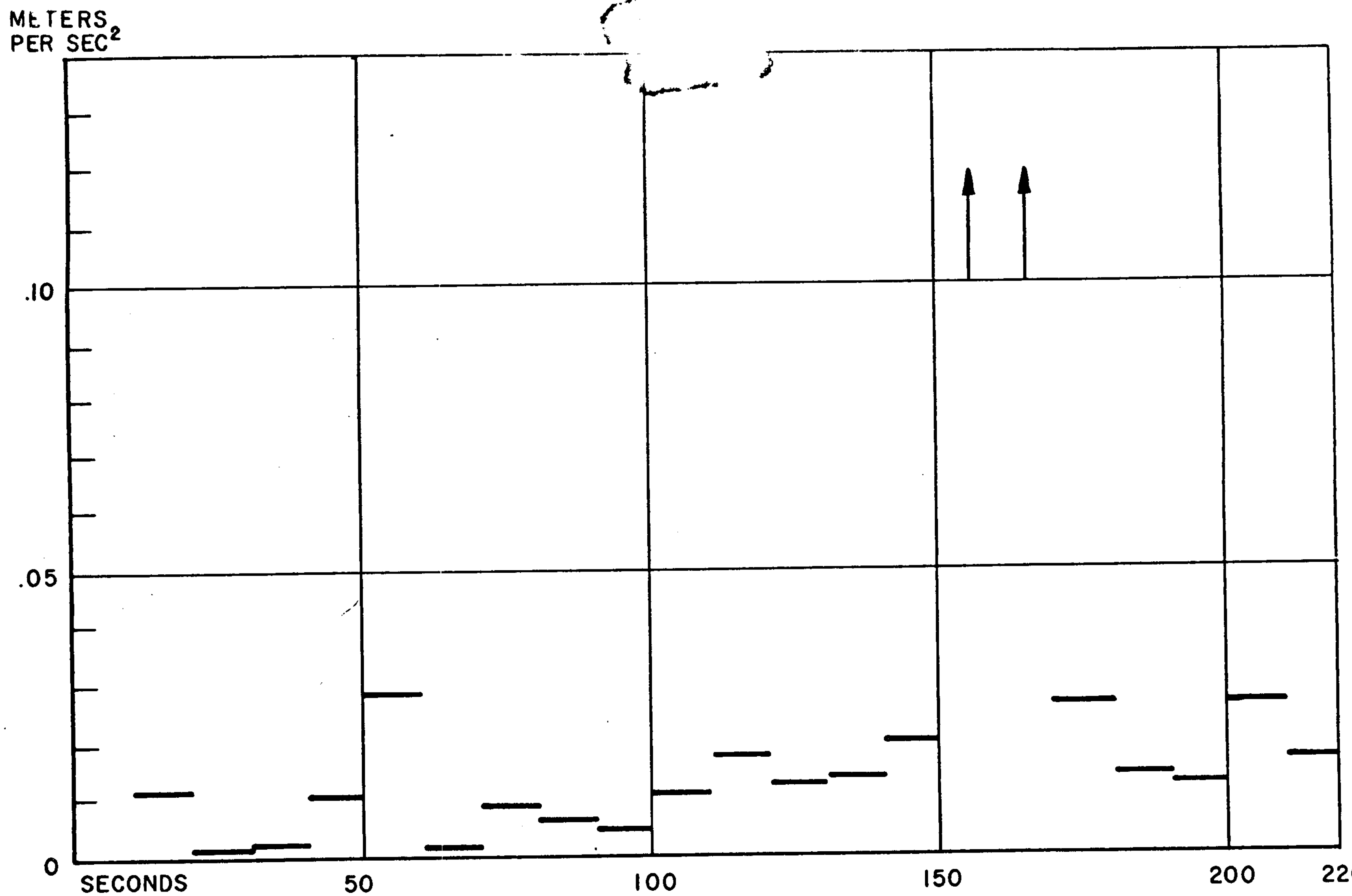

FIG 29 STANDARD DEVIATION OF SMOOTH Y ACCELERATION UDOP

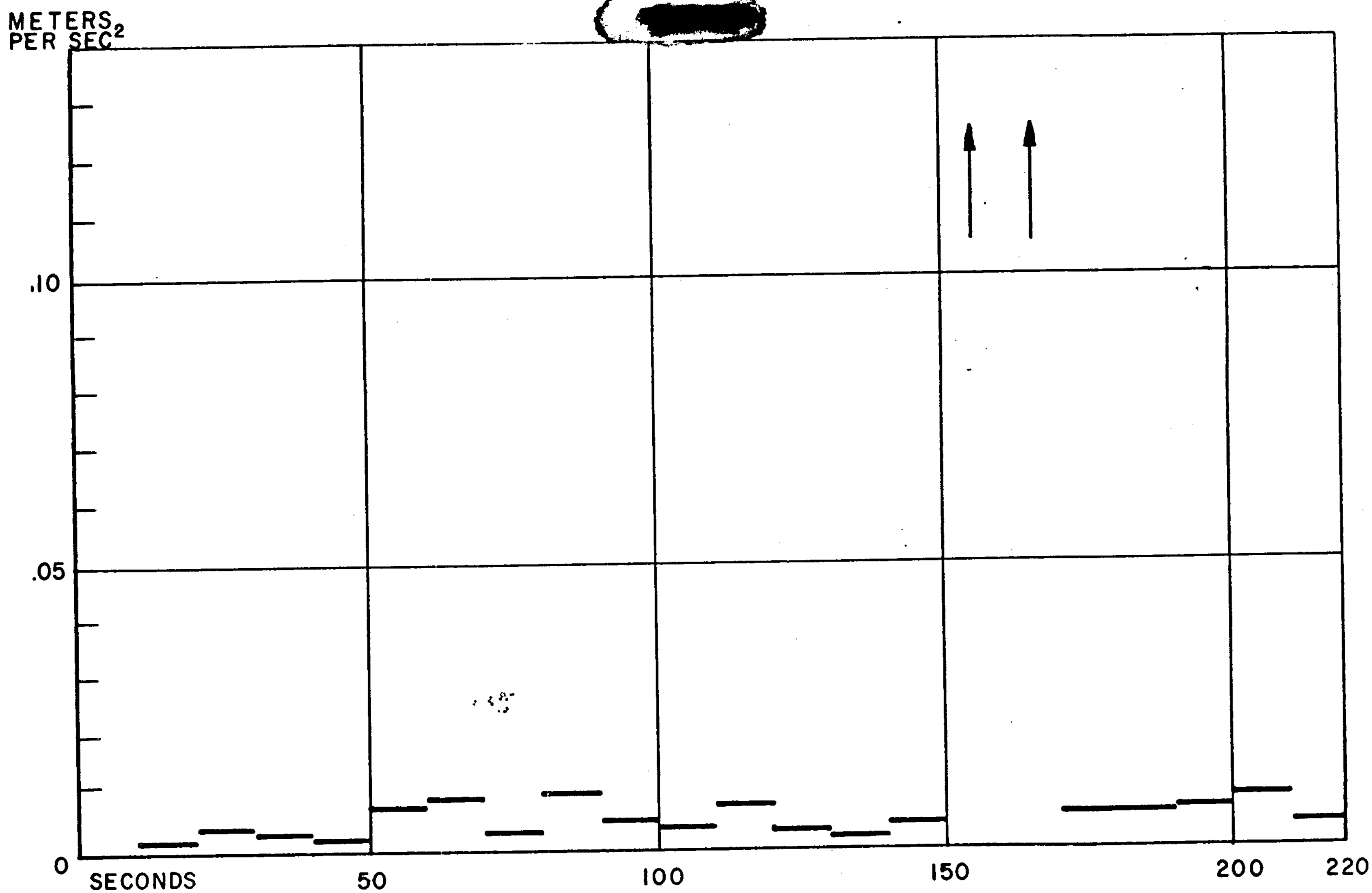

FIG 30 STANDARD DEVIATION OF SMOOTH Z ACCELERATIONS UDOP

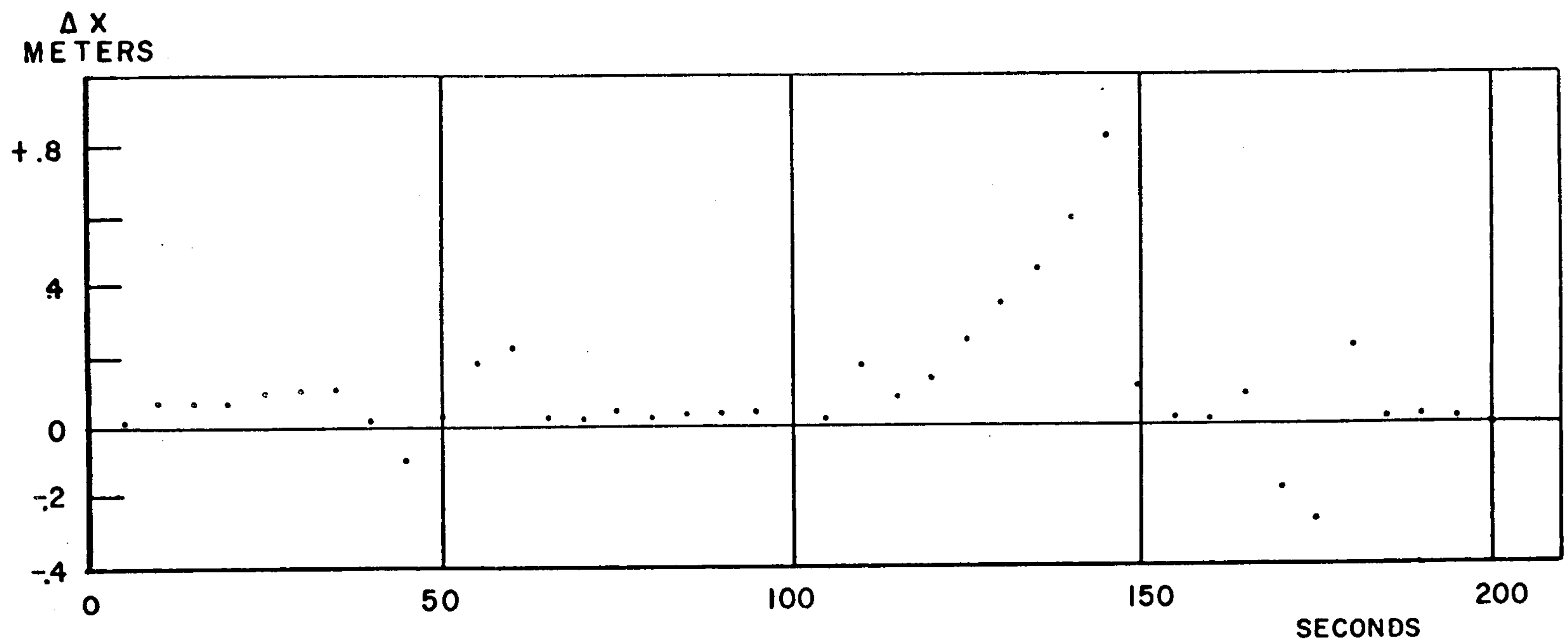

FIG 31 X POSITION ERROR DUE TO SMOOTHING

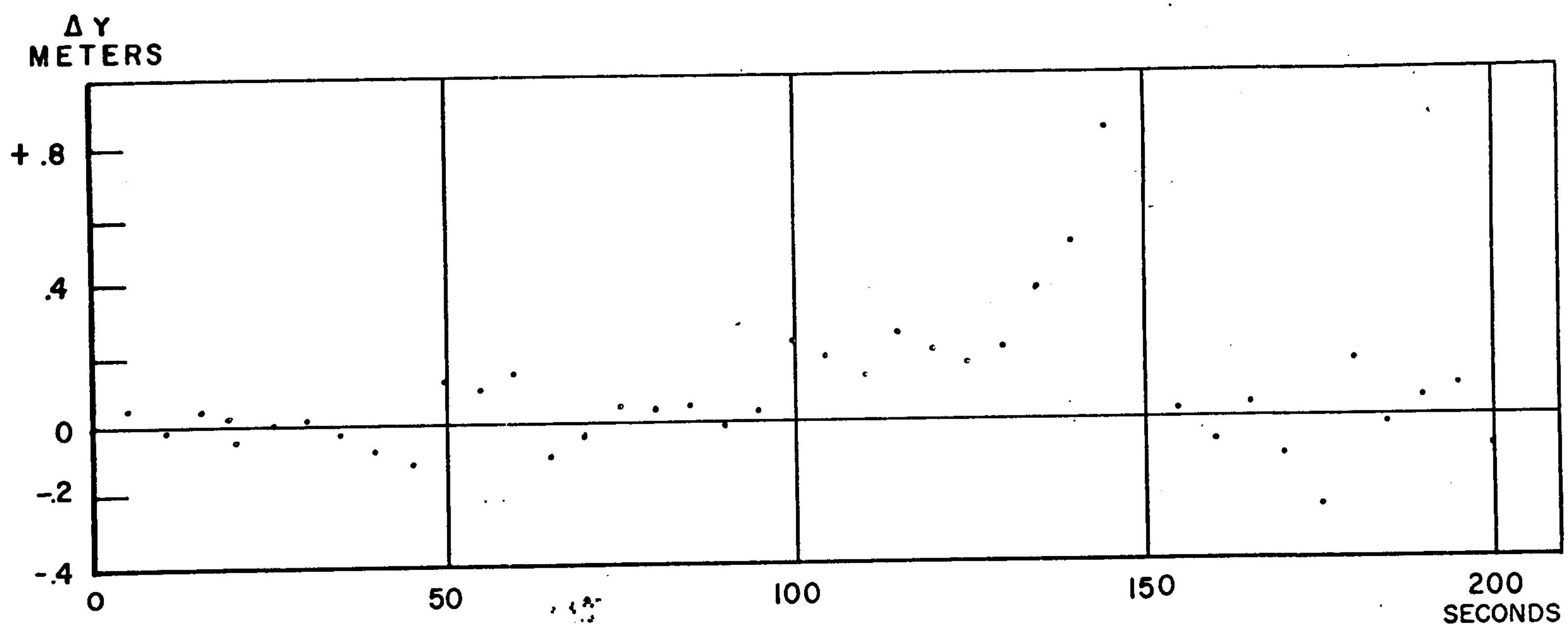

FIG 32 Y POSITION ERROR DUE TO SMOOTHING

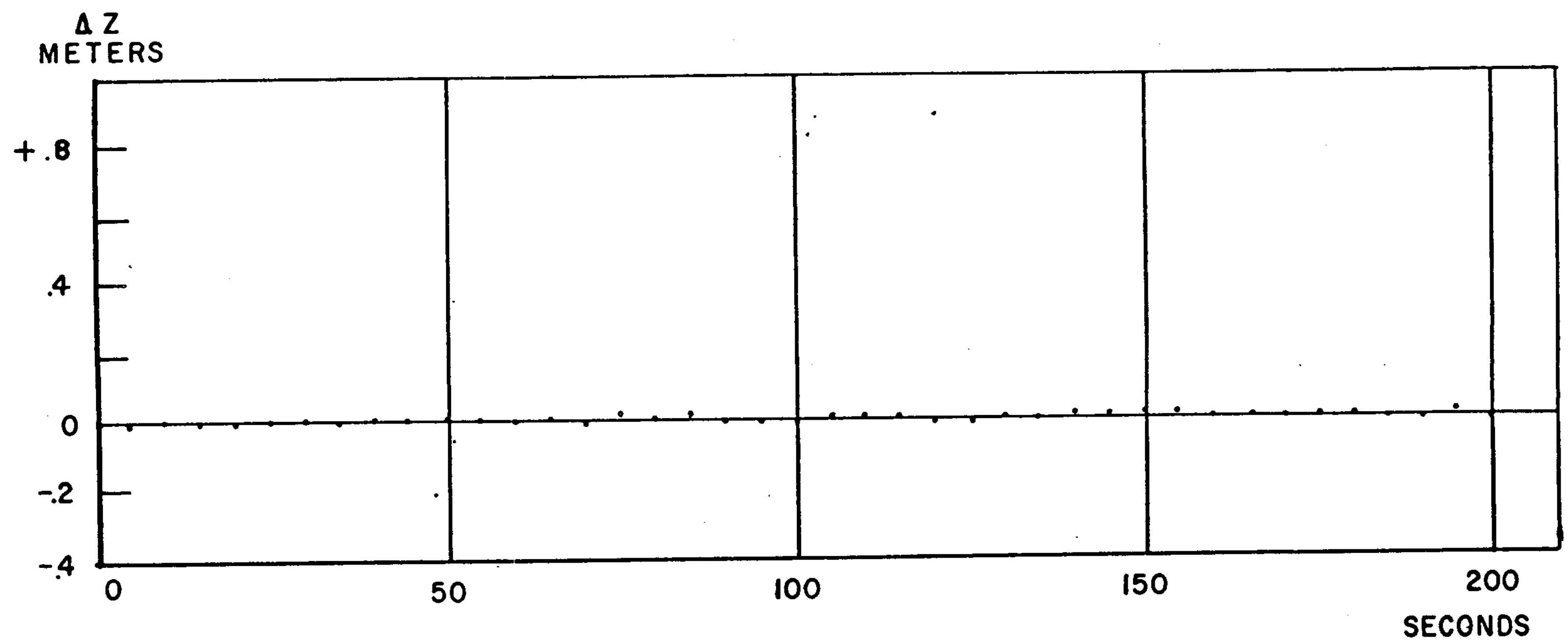

FIG 33 Z POSITION ERROR DUE TO SMOOTHING

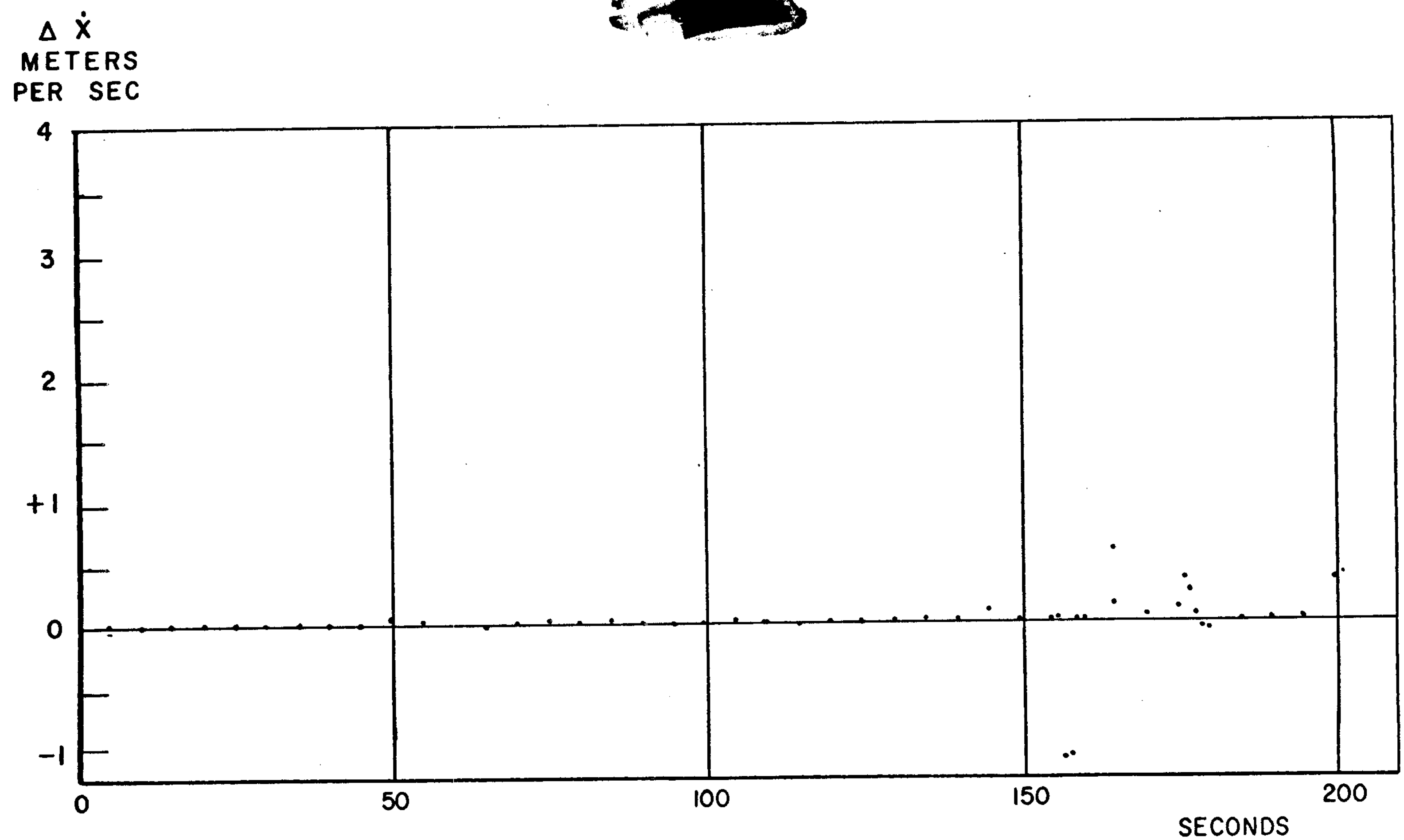

FIG 34 X VELOCITY ERROR DUE TO SMOOTHING

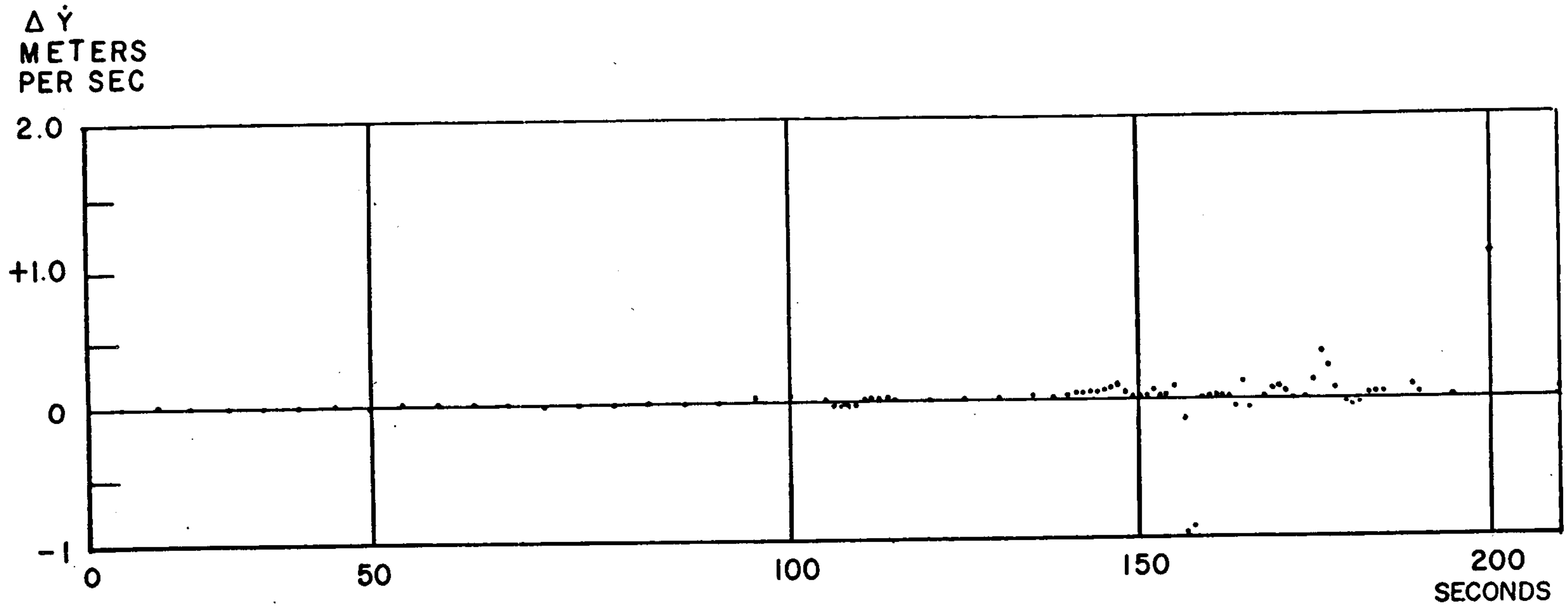

FIG 35 Y VELOCITY ERROR DUE TO SMOOTHING

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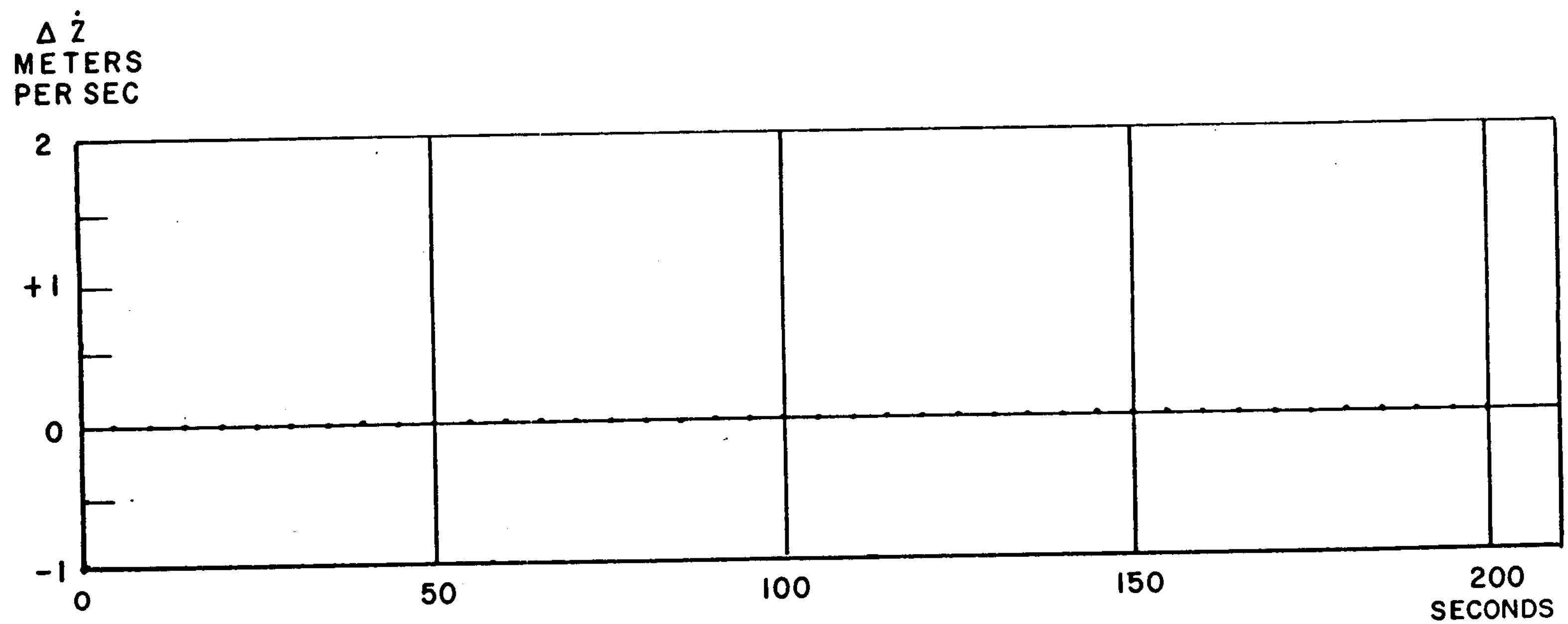

FIG 36 Z VELOCITY ERROR DUE TO SMOOTHING

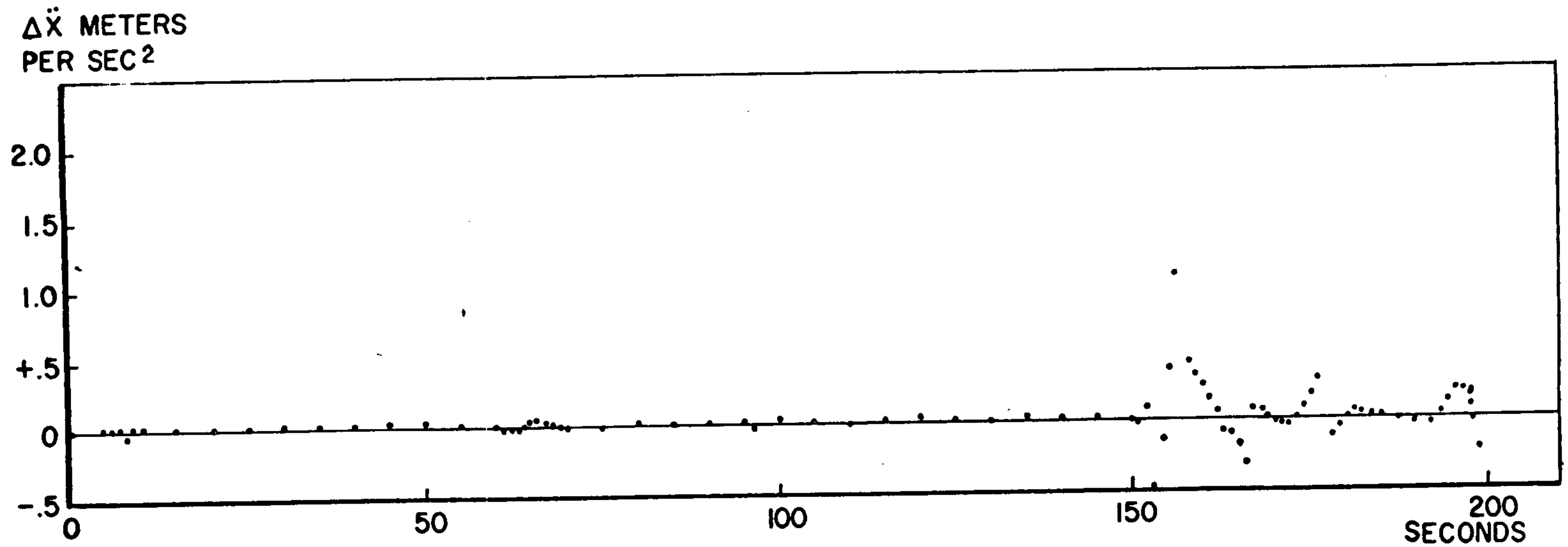

FIG. 37 X ACCELERATION ERROR DUE TO SMOOTHING

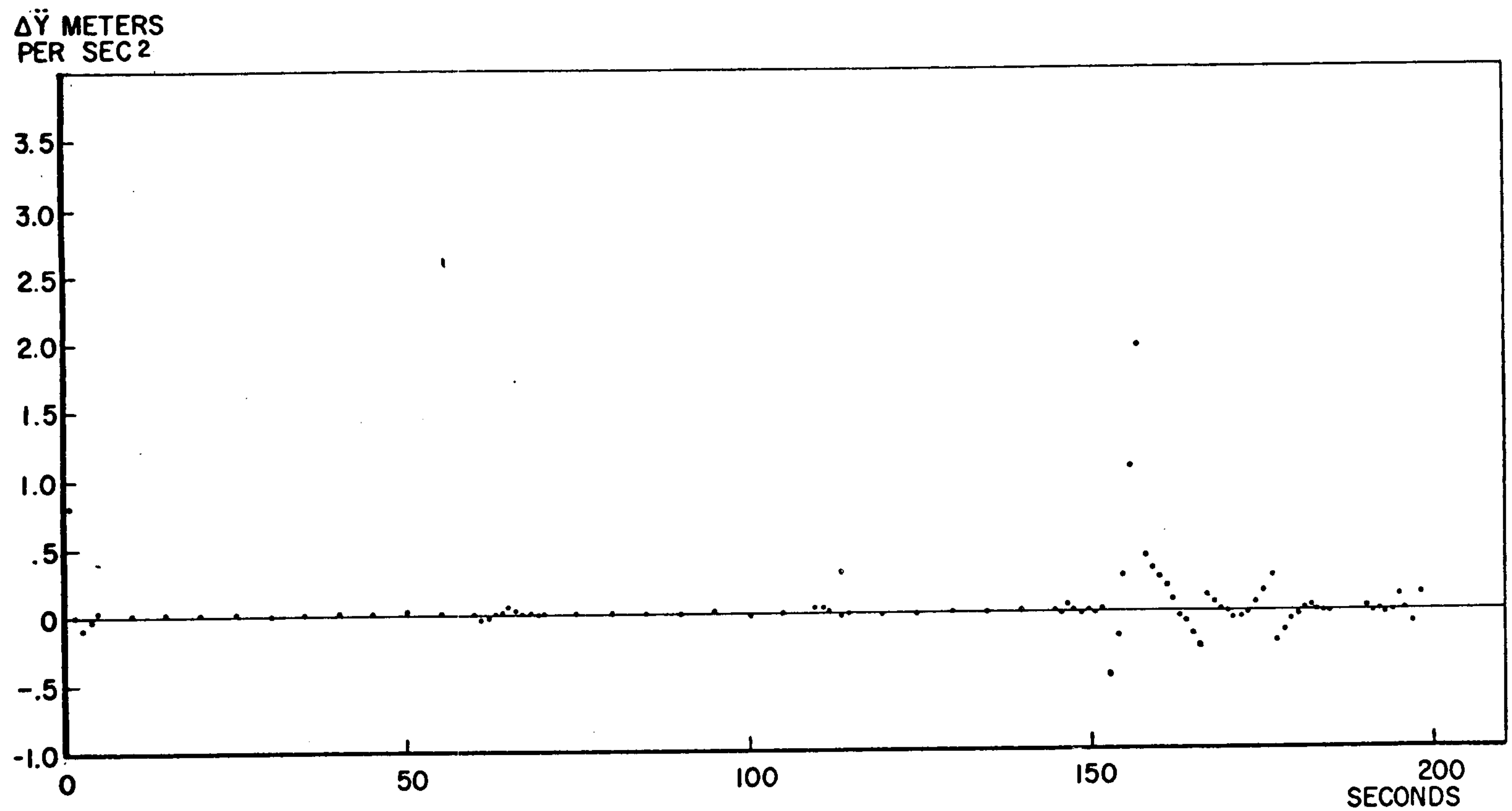

FIG. 38 Y ACCELERATION ERROR DUE TO SMOOTHING

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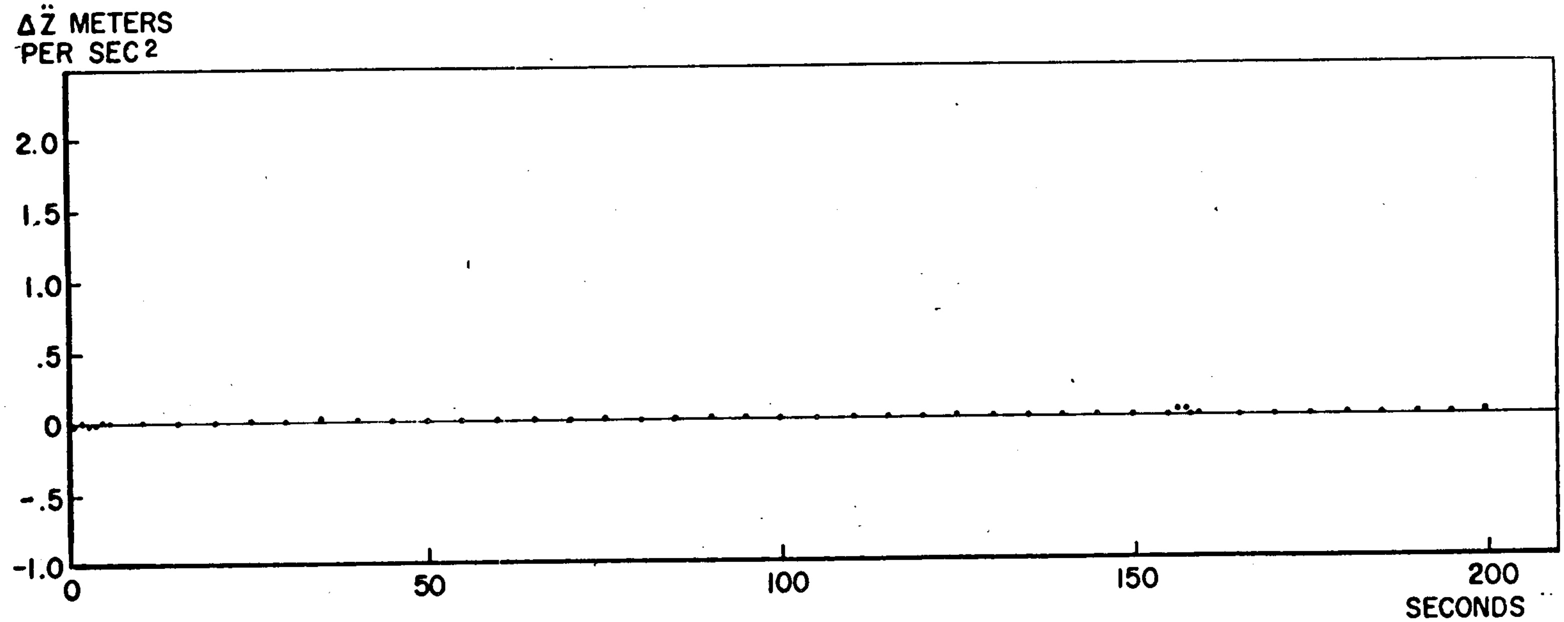

FIG. 39 Z ACCELERATION ERROR DUE TO SMOOTHING

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